Quiz 01

1. $\because \int_0^1 \frac{1}{x^p}\,dx$ diverges when $p \geq 1$

and $\int_1^\infty \frac{1}{x^p}\,dx$ diverges when $0 < p \leq 1$

$\therefore \int_0^\infty \frac{1}{x^p}\,dx$ diverges for all $p > 0$

2. Integral test : Let $a_n = f(n)$

where $f(x)$ is a continuous, positive, decreasing function of x, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^\infty f(x)\,dx$ both converge or both diverge

Ex: $a_n = \frac{1}{n^2}$

$\because f(x) = \frac{1}{x^2}$ is a continuous, positive, decreasing function

and $\int_1^\infty \frac{1}{x^2}\,dx$ converges.

$\therefore$ By Integral test, $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

3. Ratio test : If $a_n > 0$, $\forall n \in \mathbb{N}$, and suppose that

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$, then $\sum a_n$ converges if $\rho < 1$

diverges if $\rho > 1$

when $\rho = 1$, $\sum a_n$ may converge or diverge

Ex: $a_n = \frac{1}{n!}$

$\because a_n > 0, \forall n \in \mathbb{N}$ and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$

$\therefore$ By Ratio test, $\sum_{n=1}^{\infty} \frac{1}{n!}$ Converges

4. Root test : If $\exists k \in \mathbb{N}$ s.t. $a_n \geq 0, \forall n \geq k$, and suppose that

$\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$, then $\sum a_n$ converges if $\rho < 1$
diverges if $\rho > 1$

when $\rho = 1$, $\sum a_n$ may converge or diverge

Ex: $a_n = \frac{1}{2^n}$

$\because a_n > 0, \forall n \in \mathbb{N}$ and $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2} < 1$

$\therefore$ By Root test, $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges

5. $\because \lim_{n\to\infty} \frac{\frac{1}{n\sqrt[n]{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$, and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

$\therefore$ By Limit Comparison Test, $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$ diverges