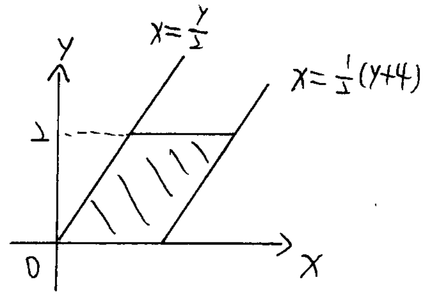
14.7-8 $\quad x = u + \frac{1}{2}v$, $y = v$ $\Rightarrow \begin{cases} \frac{\partial x}{\partial u} = 1, & \frac{\partial x}{\partial v} = \frac{1}{2} \\ \frac{\partial y}{\partial u} = 0, & \frac{\partial y}{\partial v} = 1 \end{cases} \Rightarrow J(u,v) = 1$



$$\int_0^2 \int_{\frac{y}{2}}^{\frac{y+4}{2}} y^3 (2x-y) e^{(2x-y)^2} \, dx\, dy$$

$$= \int_0^2 \int_0^2 v^3 (2u) e^{(2u)^2} \cdot 1 \, du\, dv$$

$$\begin{cases} x = \frac{y}{2} \\ x = \frac{1}{2}(y+4) \end{cases} \Rightarrow \begin{cases} u = 0 \\ u = 2 \end{cases}$$

$$= \int_0^2 \frac{1}{4} v^3 e^{4u^2} \Big|_{u=0}^{u=2} dv$$

$$= \int_0^2 \frac{1}{4}(e^{16}-1) v^3 \, dv = \frac{1}{16}(e^{16}-1) v^4 \Big|_0^2 = e^{16}-1$$

Ch14-28 $\quad I = \int_0^\infty e^{-x^2} dx$

$$\Rightarrow I^2 = \left(\int_0^\infty e^{-x^2} dx\right)\left(\int_0^\infty e^{-y^2} dy\right) = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$$

$$= \int_0^{\frac{\pi}{2}} \int_0^\infty e^{-r^2} r \, dr\, d\theta = \int_0^{\frac{\pi}{2}} \lim_{a\to\infty} \left(-\frac{1}{2} e^{-r^2} \Big|_0^a\right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2} \, d\theta = \frac{1}{2}\theta \Big|_0^{\frac{\pi}{2}} = \frac{\pi}{4}$$

$$\Rightarrow I = \int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$