13.7-36 $f(x,y)=xy+2x-\ln x^2y$, $x>0$, $y>0$

$$\Rightarrow \begin{cases} f_x(x,y)=y+2-\frac{2}{x} \\ f_y(x,y)=x-\frac{1}{y} \end{cases}$$

$$\begin{cases} y+2-\frac{2}{x}=0 \\ x-\frac{1}{y}=0 \end{cases} \Rightarrow y+2-2y=0 \Rightarrow y=2 \Rightarrow x=\frac{1}{2}$$

$\Rightarrow$ critical point : $(\frac{1}{2},2)$

$$\Rightarrow \begin{cases} f_{xx}(x,y)=\frac{2}{x^2} \\ f_{xy}(x,y)=1 \\ f_{yy}(x,y)=\frac{1}{y^2} \end{cases} \Rightarrow \left(f_{xx}f_{yy}-f_{xy}^2\right)\Big|_{(\frac{1}{2},2)}=8\cdot\frac{1}{4}-1^2=1>0$$

and $f_{xx}(\frac{1}{2},2)=8>0$

Hence f has local minimum at $(\frac{1}{2},2)$

42. $\because f_{xx}(a,b)\cdot f_{yy}(a,b)<0 \Rightarrow \left(f_{xx}f_{yy}-f_{xy}^2\right)\Big|_{(a,b)}<0$

$\Rightarrow f$ won't have extreme value at (a,b)

13.8-8 $f(x,y)=x^2+y^2$, $x^2+xy+y^2=1 \Rightarrow$ let $g(x,y)=x^2+xy+y^2-1$

$$\begin{cases} \nabla f=\lambda\nabla g \\ g=0 \end{cases} \Rightarrow \begin{cases} \begin{pmatrix} 2x \\ 2y \end{pmatrix}=\lambda\begin{pmatrix} 2x+y \\ x+2y \end{pmatrix} \\ x^2+xy+y^2-1=0 \end{cases}$$

(i) If $\lambda=0 \Rightarrow x=y=0$, but $g(0,0)\neq 0$

(ii) $\lambda\neq 0$, if $x=0$, $\because 2x=\lambda(2x+y) \Rightarrow y=0$ (X)

hence $x\neq 0$, similarly $y\neq 0$

(iii) $\lambda \neq 0,\ x \neq 0,\ y \neq 0$

$$\Rightarrow \frac{2x}{2y} = \frac{\lambda(2x+y)}{\lambda(x+2y)} \Rightarrow x^2+2xy = 2xy+y^2 \Rightarrow y = \pm x$$

① $y = x \Rightarrow x^2+xy+y^2-1=0 \Rightarrow 3x^2=1 \Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),\ \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

② $y = -x \Rightarrow x^2+xy+y^2-1=0 \Rightarrow x^2=1 \Rightarrow x = \pm 1$

$$\Rightarrow (1,-1),\ (-1,1)$$

$\because f\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = f\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) = \frac{2}{3}$ and $f(1,-1) = f(-1,1) = 2$

$\therefore$ The nearest : $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right),\ \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$
farthest : $(1,-1),\ (-1,1)$

29 $f(x,y,z) = x^2yz+1$

$z=1 \Rightarrow g_1(x,y,z) = z-1$

$x^2+y^2+z^2=10 \Rightarrow g_2(x,y,z) = x^2+y^2+z^2-10$

$$\begin{cases} \nabla f = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2 \\ g_1 = 0 \\ g_2 = 0 \end{cases} \Rightarrow \begin{cases} \begin{pmatrix} 2xyz \\ x^2z \\ x^2y \end{pmatrix} = \lambda_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} \\ z-1=0 \quad \Rightarrow z=1 \\ x^2+y^2+z^2-10=0 \end{cases}$$

$$\Rightarrow \begin{cases} 2xy = 2\lambda_2 x \quad \Rightarrow x(y-\lambda_2)=0 \\ x^2 = 2\lambda_2 y \\ x^2y = \lambda_1 + 2\lambda_2 \\ x^2+y^2 = 9 \end{cases}$$

(i) If $x=0$

$$x^2+y^2=9 \Rightarrow y=\pm 3 \Rightarrow \lambda_1=\lambda_2=0 \Rightarrow (0,\pm 3,1)$$

(ii) If $y=\lambda_2 \Rightarrow x^2=2\lambda_2 y=2y^2$

$$x^2+y^2=9 \Rightarrow y=\pm\sqrt{3} \Rightarrow x=\pm\sqrt{6} \Rightarrow \begin{cases}\lambda_1=6y-\frac{6}{y} \\ \lambda_2=\frac{3}{y}\end{cases} \quad (y=\pm\sqrt{3})$$

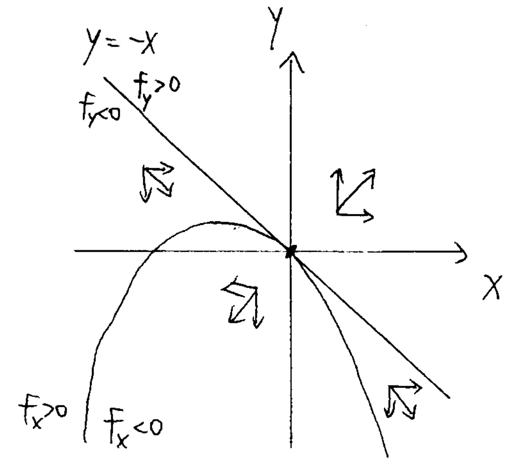
$$\Rightarrow (\sqrt{6},\sqrt{3},1),\ (\sqrt{6},-\sqrt{3},1),\ (-\sqrt{6},\sqrt{3},1),\ (-\sqrt{6},-\sqrt{3},1)$$

$\because f(0,\pm 3,1)=1$, $f(\pm\sqrt{6},\sqrt{3},1)=6\sqrt{3}+1$, $f(\pm\sqrt{6},-\sqrt{3},1)=-6\sqrt{3}+1$

$\Rightarrow$ on $\begin{cases} z=1 \\ x^2+y^2+z^2=10\end{cases}$ f has absolute maximum $1+6\sqrt{3}$ at $(\pm\sqrt{6},\sqrt{3},1)$ and absolute minimum $1-6\sqrt{3}$ at $(\pm\sqrt{6},-\sqrt{3},1)$

§13.7-extra1 $f_x(x,y) = 3x^2+2x+2y$

$f_y(x,y) = 2x+2y$



$f_x(x,y)=0 \Rightarrow 3x^2+2x+2y=0$

$\Rightarrow y = -\frac{3}{2}(x+\frac{1}{3})^2+\frac{1}{6}$

$f_y(x,y)=0 \Rightarrow 2x+2y=0$

$\Rightarrow y=-x$

By gradient analysis, $(0,0)$ is a saddle point

5/3.8-extra1

$$\begin{cases} x^2+2y^2+3z^2=6 \\ x+y+z=3 \end{cases} \quad \text{at } (1,1,1)$$

let $f(x,y,z)=x^2+2y^2+3z^2-6$

$g(x,y,z)=x+y+z-3$

$\Rightarrow \nabla f(x,y,z)=(2x,4y,6z) \Rightarrow \nabla f(1,1,1)=(2,4,6)$

$\nabla g(x,y,z)=(1,1,1) \qquad \nabla g(1,1,1)=(1,1,1)$

$\Rightarrow [\nabla f(1,1,1)]\times[\nabla g(1,1,1)]=(-2,4,-2)$

$\Rightarrow$ equation: $-2(x-1)+4(y-1)-2(z-1)=0$