Homework for week 3.

9.5-4.

$\because \lim_{n\to\infty} \frac{10^n}{n^{10}} = \infty$

$\therefore \lim_{n\to\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$ does not exist

$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$ diverge by nth-term test

8.

(i) $\ln(1+\frac{1}{n}) \geq 0, \quad \forall n \geq 1$

(ii) $\ln(1+\frac{1}{n+1}) < \ln(1+\frac{1}{n}), \quad \forall n \geq 1$

(iii) $\lim_{n\to\infty} \ln(1+\frac{1}{n}) = 0.$

$\Rightarrow \sum_{n=1}^{\infty} (-1)^n \ln(1+\frac{1}{n})$ converge by Alternating series Test

24.

$\because \lim_{n\to\infty} \sqrt[n]{10} = 1.$

$\therefore \lim_{n\to\infty} (-1)^{n+1} (\sqrt[n]{10})$ does not exist

$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} (\sqrt[n]{10})$ diverges by nth-term test.

9.6-28.

$a_n = \frac{x^n}{n\ln n}$

$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{|x|}{\sqrt[n]{n}\cdot\sqrt[n]{\ln n}} = |x|$

$\Rightarrow \sum_{n=1}^{\infty} a_n$ converges absolutely for $|x|<1$ by Root test.

if $x=1 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n\ln n}$ diverge

if $x=-1 \Rightarrow$ (i) $\frac{1}{n\ln n} \geq 0$, $\forall n \geq 1$

(ii) $\frac{1}{(n+1)\ln(n+1)} \leq \frac{1}{n\ln n}$, $\forall n \geq 1$.

(iii) $\lim_{n\to\infty} \frac{1}{n\ln n} = 0$

Hence $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\ln n}$ converges

(a) $-1 \leq x < 1$, (b) $-1 < x < 1$ (c) $x = -1$.

44.

$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$

$\xrightarrow{\text{diff. } x} \sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2} \xrightarrow{\text{mult. } x} \sum_{n=1}^{\infty} n x^n = \frac{x}{(1-x)^2}$

$\xrightarrow{\text{diff. } x} \sum_{n=1}^{\infty} n^2 x^{n-1} = \frac{(1+x)}{(1-x)^2} \xrightarrow{\text{mult. } x} \sum_{n=1}^{\infty} n^2 x^n = \frac{x(1+x)}{(1-x)^2}$

$\xrightarrow{x=\frac{1}{2}} \sum_{n=1}^{\infty} \frac{n^2}{2^n} = 6$.

§9.6 – extra 1

(i) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$: Interval of convergence is $[1,3)$

(ii) $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{n}$: Interval of convergence is $(1,3)$

(iii) $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^{2n}}{n}$: Interval of convergence is $[1,3]$

(iv) $\sum_{n=1}^{\infty} \frac{(2-x)^n}{n}$: Interval of convergence is $(1,3]$