9.4-28 $a_1 = 1$, $a_{n+1} = \dfrac{1+\tan^{-1} n}{n} a_n$

$\because a_n > 0$, $\forall n \in \mathbb{N}$ and $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \dfrac{1+\tan^{-1} n}{n} = 0 < 1$

By ratio test, $\sum\limits_{n=1}^{\infty} a_n$ converges

34. $a_1 = \frac{1}{2}$, $a_{n+1} = \dfrac{n+\ln n}{n+10} a_n$

$\because \lim\limits_{n\to\infty} \ln n = \infty$, $\therefore \exists K \in \mathbb{N}$ s.t. $\ln n > 10$, $\forall n \geq K$

$\Rightarrow a_{n+1} = \dfrac{n+\ln n}{n+10} a_n \geq a_n$, $\forall n \geq K$

$\Rightarrow \lim\limits_{n\to\infty} a_n \geq a_K \neq 0 \Rightarrow \sum\limits_{n=1}^{\infty} a_n$ diverges

40 $\sum\limits_{n=1}^{\infty} \dfrac{n^n}{n!} \Rightarrow \lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \dfrac{(n+1)^{n+1}}{(n+1)!} \cdot \dfrac{n!}{n^n} = \lim\limits_{n\to\infty} \left(\dfrac{n+1}{n}\right)^n$

$= \lim\limits_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e^1 > 1$

By ratio test, $\sum\limits_{n=1}^{\infty} \dfrac{n^n}{n!}$ diverges

44. (i) $\lim\limits_{n\to\infty} \dfrac{a_{n+1}}{a_n} = \lim\limits_{n\to\infty} \dfrac{1}{(\ln(n+1))^p} \cdot (\ln n)^p = \lim\limits_{n\to\infty} \left(\dfrac{\ln n}{\ln(n+1)}\right)^p = 1$

(ii) $\lim\limits_{n\to\infty} \sqrt[n]{a_n} = \lim\limits_{n\to\infty} \dfrac{1}{(\ln n)^{\frac{p}{n}}} = \lim\limits_{n\to\infty} (\ln n)^{-\frac{p}{n}} = \lim\limits_{n\to\infty} e^{\ln[(\ln n)^{-\frac{p}{n}}]}$

$= \lim\limits_{n\to\infty} e^{-\frac{p}{n}\ln(\ln n)} = \lim\limits_{n\to\infty} e^{-\frac{\frac{1}{\ln n}\cdot\frac{1}{n}}{1}\cdot p} = 1$

(iii) $\lim\limits_{n\to\infty} \dfrac{\frac{1}{(\ln n)^p}}{\frac{1}{n}} = \lim\limits_{n\to\infty} \dfrac{n}{(\ln n)^p} = \lim\limits_{n\to\infty} \dfrac{1}{p(\ln n)^{p-1}(\frac{1}{n})} = \lim\limits_{n\to\infty} \dfrac{n}{p(\ln n)^{p-1}} = \infty$, $\forall p \in \mathbb{R}$

$\because \sum\limits_{n=2}^{\infty} \frac{1}{n}$ diverges, $\therefore \sum\limits_{n=2}^{\infty} \dfrac{1}{(\ln n)^p}$ diverges.