1. $\frac{dy}{dx} + xy = x \ , \ y(0) = -6$

$\Rightarrow \frac{dy}{dx} = x - xy = x(1-y)$

$\Rightarrow \int \frac{1}{1-y}\, dy = \int x\, dx \Rightarrow -\ln|1-y| = \frac{x^2}{2} + C$

$y(0) = -6 \Rightarrow -\ln(1-(-6)) = 0 + C \Rightarrow C = -\ln 7$

$\Rightarrow y = 1 - 7e^{-\frac{x^2}{2}}$

2. (i) $\int_0^3 2x\sqrt{9-x^2}\, dx = -\frac{2}{3}(9-x^2)^{\frac{3}{2}} \Big|_0^3 = \frac{2}{3} \cdot 9^{\frac{3}{2}} = 18$

(ii) $\int_{-3}^{3} \frac{1}{2}\left(\sqrt{9-y^2}\right)^2 dy = \frac{1}{2}\int_{-3}^{3} 9 - y^2\, dy = \frac{1}{2}\left(9y - \frac{y^3}{3}\right)\Big|_{-3}^{3}$

$= \frac{1}{2}(18 - (-18)) = 18$

3. $\text{Volume} = \int_0^1 \pi\, (f(y))^2\, dy$

$\text{Area} = \int_0^1 2\pi x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_0^1 2\pi f(y)\sqrt{1+\left(\frac{df}{dy}\right)^2}\, dy$

4. (i) $\lim_{x\to\infty} \tanh x = \lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x\to\infty} \frac{1-e^{-2x}}{1+e^{-2x}} = 1$

(ii) $\frac{d}{dx}\tanh x = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right) = \frac{(\cosh x)^2 - (\sinh x)^2}{(\cosh x)^2} = \frac{\frac{e^{2x}+2+e^{-2x}}{4} - \frac{e^{2x}-2+e^{-2x}}{4}}{(\cosh x)^2}$

$= \frac{1}{(\cosh x)^2} = \operatorname{sech}^2 x$

(iii) $\int \tanh x\, dx = \int \frac{\sinh x}{\cosh x}\, dx$, let $u = \cosh x$, $du = \sinh x\, dx$

$= \int \frac{1}{u}\, du = \ln|u| + C = \ln \cosh x + C$