1. Fundamental Theorem of Calculus, Part 1.

If f is continuous on $[a,b]$, then $F(x)=\int_a^x f(t)dt$ has a derivative at every point of $[a,b]$ and

$$\frac{dF}{dx}=\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Part 2

If f is continuous on $[a,b]$ and F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)dx = F(b)-F(a)$

2. $y''(x)=1+x$ with $y'(0)=1$

$$\Rightarrow y'(x)=\int_0^x y''(t)dt+1=\int_0^x 1+t\,dt+1=\frac{x^2}{2}+x+1$$

with $y(1)=2$

$$\Rightarrow y(x)=\int_1^x y'(t)dt+2=\int_1^x \frac{t^2}{2}+t+1\,dt+2=\left(\frac{x^3}{6}+\frac{x^2}{2}+x\right)-\left(\frac{1}{6}+\frac{1}{2}+1\right)+2$$

$$=\frac{x^3}{6}+\frac{x^2}{2}+x+\frac{1}{3}$$

3. $y=x^x=e^{\ln x^x}=e^{x\ln x}$, $x>0$

$$\Rightarrow \frac{dy}{dx}=e^{x\ln x}\cdot\frac{d}{dx}(x\ln x)=x^x\cdot(\ln x+1)$$

4. $\frac{d}{dy} f^{-1}(y)\Big|_{y=2} = \dfrac{1}{\frac{d}{dx} f(x)\Big|_{x=1}}$ $\quad (\because f(1)=2)$

$= \dfrac{1}{1 \cdot 1}$

5. $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$ $\quad$ (let $u = \sin x$, $du = \cos x \, dx$)

$= \int \frac{1}{u} du = \ln|u| + c$

$= \ln|\sin x| + C$, for some constant C