1.

$$\frac{d^4}{dx^4}\left(x^4\cos(x-1)\right)\Big|_{x=1}$$

$$= 24\cos(x-1) + 4\cdot\left[24x(-\sin(x-1))\right]$$

$$+6\left[12x^2(-\cos(x-1))\right] + 4\left[4x^3(\sin(x-1))\right] + x^4\cos(x-1)\Big|_{x=}$$

$$= 24 + 0 - 72 + 0 + 1 = -47 \quad \#$$

2. $y = \sin^2(\tan(x^3))$

$\Rightarrow \frac{dy}{dx} = 2\sin(\tan(x^3)) \cdot \cos(\tan(x^3)) \cdot \sec^2(x^3) \cdot 3x^2$

3. $x^4 + y^4 = 2$

$\Rightarrow \quad 4x^3 + 4y^3 y' = 0$

$\Rightarrow \quad x^3 + y^3 y' = 0 \quad \Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = -1$

$3x^2 + 3y^2(y')^2 + y^3 y'' = 0 \quad \Rightarrow \quad \left.\frac{dy}{dx}\right|_{(x,y)=(1,1)} = -6$

4. Let $f(x) = (1+x)^{\frac{1}{2}} \Rightarrow (1.001)^{\frac{1}{2}} = f(0.001)$

Linear approximation of $f(x)$ near $x=0$: $L(x) = 1 + \frac{1}{2}x$

Approximate value of $\sqrt{1.001} \approx L(0.001) = 1.0005$

Since $f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}}$

$$|f(0.001) - L(0.001)| \leq \frac{1}{2} \max_{\xi \in [0, 0.001]} |f''(\xi)| (0.001 - 0)^2 = \frac{1}{2} \cdot \frac{1}{4} \cdot 10^{-6} = \frac{1}{8} \cdot 10^{-6}$$

Hence the error is smaller than 10^{-6}

5 For $x \neq 1$, $\dfrac{f(x)-f(1)}{x-1} = 2 + \alpha(x)$

$\because \lim_{x\to 1} \alpha(x) = 0$

$\therefore \lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1} = \lim_{x\to 1} (2+\alpha(x)) = 2$

$\Rightarrow f(x)$ is diff. at $x=1$.