7.4-46

The area swept out by AB

$$= \int_a^{a+h} 2\pi \sqrt{r^2-x^2} \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\, dx \qquad \left(y=\sqrt{r^2-x^2} \Rightarrow \frac{dy}{dx} = \frac{-2x}{2\sqrt{r^2-x^2}} = \frac{x}{\sqrt{r^2-x^2}} \right)$$

$$= \int_a^{a+h} 2\pi \sqrt{(r^2-x^2)\left(1+\frac{x^2}{r^2-x^2}\right)}\, dx$$

$$= \int_a^{a+h} 2\pi \sqrt{r^2-x^2+x^2}\, dx = \int_a^{a+h} 2\pi r\, dx = 2\pi r \int_a^{a+h} 1\, dx$$

$$= 2\pi r\, [x]_a^{a+h} = \underbrace{2\pi r}_{\text{constant}} h$$

Hence, the area does not depend on $\underbrace{\text{the location of the interval}}_{\Rightarrow a}$

but only depend on $\underbrace{\text{the length of the interval}}_{\Rightarrow h}$

7.8-4 (a)

$$\int_0^{\pi} \frac{\sqrt{3}}{4}\left(2\sqrt{\sin x}\right)^2 dx = \sqrt{3}\int_0^{\pi} \sin x\, dx = \sqrt{3}\left(-\cos x\right)\Big|_0^{\pi} = 2\sqrt{3}$$

(b)

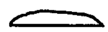
$$\int_0^{\pi} \left(2\sqrt{\sin x}\right)^2 dx = 4\int_0^{\pi} \sin x\, dx = 8$$

10.

$$\int_{-1}^{1} \frac{1}{2}\cdot\left(2\sqrt{1-y^2}\right)^2 dy = \int_{-1}^{1} 2(1-y^2)\,dy = 2\left(y-\frac{y^3}{3}\right)\Big|_{-1}^{1} = \frac{8}{3}$$

7.8-example2

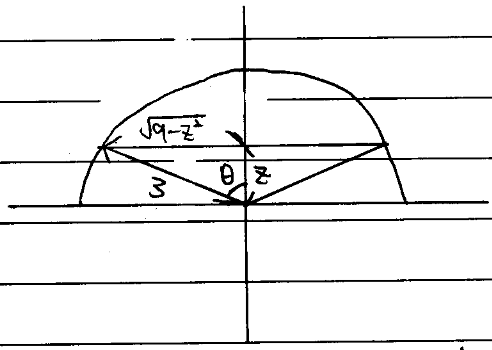
Slicing in the z-direction

$A(z) =$  $-$  $=$ 

$= \pi \cdot 3^2 \cdot \frac{2\theta}{2\pi} - \frac{1}{2} \cdot z \cdot 2\sqrt{9-z^2}$

$= 9\cos^{-1}\left(\frac{z}{3}\right) - z\sqrt{9-z^2}$

At height z



$\Rightarrow \cos\theta = \frac{z}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{z}{3}\right)$

$\text{Volume} = \int_0^3 9\cos^{-1}\left(\frac{z}{3}\right) - z\sqrt{9-z^2}\, dz$

(i) $\int z\sqrt{9-z^2}\, dz$, let $u = 9-z^2$, $du = -2z\, dz$

$= \int -\frac{1}{2}\sqrt{u}\, du = -\frac{1}{3}u^{\frac{3}{2}} + C = -\frac{1}{3}(9-z^2)^{\frac{3}{2}} + C$

(ii) $\int \cos^{-1}\left(\frac{z}{3}\right) dz$, let $w = \cos^{-1}\left(\frac{z}{3}\right) \Rightarrow \cos w = \frac{z}{3} \Rightarrow -\sin w\, dw = \frac{1}{3} dz$

$= \int -3w\sin w\, dw$, let $\hat{u} = w$, $dv = \sin w\, dw$

$\Rightarrow d\hat{u} = dw$, $v = -\cos w$

$= -3\left(-w\cos w + \int \cos w\, dw\right)$

$= 3w\cos w - 3\sin w + C = z\cos^{-1}\left(\frac{z}{3}\right) - \sqrt{9-z^2} + C$

$\text{Volume} = 9\left(z\cos^{-1}\left(\frac{z}{3}\right) - \sqrt{9-z^2}\right)\Big|_0^3 + \frac{1}{3}(9-z^2)^{\frac{3}{2}}\Big|_0^3$

$= 9(0-(-3)) + \left(0 - \frac{1}{3}\cdot 9^{\frac{3}{2}}\right) = 27 - 9 = 18$