6.2-32

$f(x) = x^2 - 4x - 5, \quad x > 2$

$$\left.\frac{df^{-1}}{dx}\right|_{x=0} = \frac{1}{\left.\frac{df}{dx}\right|_{x=5}} = \frac{1}{\left.2x-4\right|_{x=5}} = \frac{1}{6}$$

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If f and g are one-to-one $\Rightarrow$ $f \circ g$ is one-to-one

Since if $x \neq y \Rightarrow g(x) \neq g(y)$ (g is one-to-one)

$\Rightarrow f(g(x)) \neq f(g(y))$ (f is one-to-one)

46. If g is not one-to-one

$\Rightarrow \exists\ x \neq y$ s.t. $g(x) = g(y) \Rightarrow f(g(x)) = f(g(y))$

$\Rightarrow f \circ g$ is not one-to-one ($\rightarrow\leftarrow$)

Hence g must be one-to-one

6.3-50

$y = \frac{\theta \cdot \sin\theta}{\sqrt{\sec\theta}} \Rightarrow \ln y = \ln\theta + \ln\sin\theta - \frac{1}{2}\ln\sec\theta$

$\Rightarrow \frac{1}{y}\frac{dy}{d\theta} = \frac{1}{\theta} + \frac{\cos\theta}{\sin\theta} - \frac{\sec\theta\tan\theta}{2\sec\theta}$

$\Rightarrow \frac{dy}{d\theta} = \frac{\theta\sin\theta}{\sqrt{\sec\theta}}\left(\frac{1}{\theta} + \frac{\cos\theta}{\sin\theta} - \frac{\tan\theta}{2}\right)$

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$3^{\log_3(x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10}(2)}$

$\Rightarrow x^2 = 5x - 3 \cdot 2 = 5x - 6$

$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0$

$\Rightarrow x = 2$ or $x = 3$

6.2-28 (c)

$f(x) = 2x^2,\ x \geq 0,\ a = 5 \Rightarrow f(a) = 50$

$f'(x) = 4x \Rightarrow f'(5) = 20$

$f^{-1}(x) = \sqrt{\frac{x}{2}} \Rightarrow \frac{df^{-1}(x)}{dx} = \frac{1}{2\sqrt{\frac{x}{2}}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{2x}} \Rightarrow \frac{df^{-1}}{dx}(50) = \frac{1}{20} = \frac{1}{f'(5)}$

6.3 32. $\frac{dy}{dt} = \frac{-\ln t}{t^2}$

34. $\frac{dy}{dx} = \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$

36. $\frac{dy}{dt} = \frac{\cos t - \sin t}{\sin t}$

38. $\frac{dy}{d\theta} = \frac{1}{2\theta(1+\sqrt{\theta})}$

40. $\frac{dy}{d\theta} = \sec\theta$

86. $\frac{dy}{dx} = x^{(x+1)}\left(\frac{x+1}{x} + \ln x\right)$

88. $\frac{dy}{dt} = t^{\sqrt{t}}\left(\frac{\ln t}{2\sqrt{t}} + \frac{1}{\sqrt{t}}\right)$

90. $\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + (\cos x)\ln x\right)$

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$$\lim_{n\to\infty} \sum_{k=\frac{n}{2}}^{n} \frac{1}{k} = \lim_{n\to\infty} \sum_{k=\frac{n}{2}}^{n} \frac{1}{\frac{k}{n}} \cdot \frac{1}{n}$$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{x}\,dx = \ln|x|\Big]_{\frac{1}{2}}^{1} = 0 - \ln\frac{1}{2} = \ln 2$$