5.3-36

$$\frac{2}{5} \le \int_0^{0.5} \frac{1}{1+x^2}\,dx \le \frac{1}{2} \quad \text{and} \quad \frac{1}{4} \le \int_{0.5}^{1} \frac{1}{1+x^2}\,dx \le \frac{2}{5}$$

$$\Rightarrow \frac{2}{5} + \frac{1}{4} \le \int_0^{0.5} \frac{1}{1+x^2}\,dx + \int_{0.5}^{1} \frac{1}{1+x^2}\,dx \le \frac{1}{2} + \frac{2}{5}$$

$$\Rightarrow \frac{13}{20} \le \int_0^1 \frac{1}{1+x^2}\,dx \le \frac{9}{10}$$

40

Let $h(x) = f(x) - g(x) \Rightarrow \frac{1}{b-a}\int_a^b h(x)\,dx = av(h) = h(c)$ for some $c \in [a,b]$

$\Rightarrow h(c) = \frac{1}{b-a}\int_a^b f(x) - g(x)\,dx = 0$ for some $c \in [a,b]$

$\Rightarrow f(c) = g(c)$ for some $c \in [a,b]$

5.4-69 $\int_1^x f(t)dt = x^2-2x+1$

$f(x) = \frac{d}{dx}\left(\int_1^x f(t)dt\right) = \frac{d}{dx}(x^2-2x+1) = 2x-2$

73

a) f has a positive derivative for all values of x

$\Rightarrow f$ is continuous on $\mathbb{R}$

$\Rightarrow g$ is a differentiable function of x (Fundamental Theorem)

b)

By a) $\Rightarrow g$ is a continuous function of x

c)

$g'(x) = f(x) \Rightarrow g'(1) = f(1) = 0$

d), e), f)

$g''(x) = f'(x) \Rightarrow g''(1) = f'(1) > 0$ (g has no inflection at $x=1$)

$\Rightarrow g$ has local minimum at $x=1$

g)

Since $g'(1)=0$ & $g''(x) = f'(x) > 0 \Rightarrow g'$ is increasing on $\mathbb{R}$

Hence $\frac{dg}{dx} = g'(x)$ crosses the x-axis at $x=1$

5.5-36

~~$\int 2+\tan^2\theta\, d\theta = \int 2+(\sec^2\theta-1)\, d\theta = \int 1+\sec^2\theta\, d\theta$~~

~~$= \theta + \tan\theta + C$~~

55. Differential equation: $\frac{d^3y}{dx^3} = 6 + \sin x$

Initial conditions: $\frac{d^2y}{dx^2} = -1$, $\frac{dy}{dx} = -5$ and $y = 2$ when $x = 0$

$\frac{d^3y}{dx^3} = 6 + \sin x \Rightarrow \frac{d^2y}{dx^2} = \int 6 + \sin x \, dx = 6x - \cos x + C$

$\frac{d^2y}{dx^2} = -1$, when $x = 0 \Rightarrow 0 - 1 + C = -1 \Rightarrow C = 0 \Rightarrow \frac{d^2y}{dx^2} = 6x - \cos x$

$\Rightarrow \frac{dy}{dx} = \int 6x - \cos x \, dx = 3x^2 - \sin x + C$

$\frac{dy}{dx} = -5$ when $x = 0 \Rightarrow 0 - 0 + C = -5 \Rightarrow C = -5 \Rightarrow \frac{dy}{dx} = 3x^2 - \sin x - 5$

$\Rightarrow y = \int 3x^2 - \sin x - 5 \, dx = x^3 + \cos x - 5x + C$

$y = 2$ when $x = 0 \Rightarrow 0 + 1 - 0 + C = 2 \Rightarrow C = 1 \Rightarrow y = x^3 + \cos x - 5x + 1$

5.6-56 $\int_1^4 \frac{1}{2\sqrt{y}(1+\sqrt{y})^2}dy$ (let $z=1+\sqrt{y} \Rightarrow dz=\frac{1}{2\sqrt{y}}dy$)

$= \int_2^3 \frac{1}{z^2}dz = -z^{-1}\Big|_2^3 = \frac{1}{6}$

58 $\int_{\pi}^{\frac{3\pi}{2}} \tan^{-5}(\frac{\theta}{6})\sec^2(\frac{\theta}{6})\,d\theta$ (let $y=\tan(\frac{\theta}{6}) \Rightarrow dy=\frac{1}{6}\sec^2(\frac{\theta}{6})\,d\theta$)

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{6}{y^5}dy = -\frac{3}{2}y^{-4}\Big|_{\frac{1}{\sqrt{3}}}^{1} = 12$

67

a) $\frac{d(\sin^2 x + C_1)}{dx} = 2\sin x \cdot \cos x$

b) $\frac{d(-\cos^2 x + C_2)}{dx} = -2\cos x(-\sin x) = 2\sin x\cos x$

c) $\frac{d(-\frac{\cos 2x}{2} + C_3)}{dx} = \frac{\sin 2x}{2}\cdot 2 = \sin 2x = 2\sin x\cos x$

All of a), b) and c) are antiderivatives of $2\sin x\cos x$, all three integrations are correct

5.5-extra | Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\sin(\frac{k}{n})$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\sin(\frac{k}{n}) = \int_0^1 \sin x \, dx = (-\cos x)\Big|_0^1 = (-\cos 1) - (-\cos 0)$$

$$= 1 - \cos 1$$