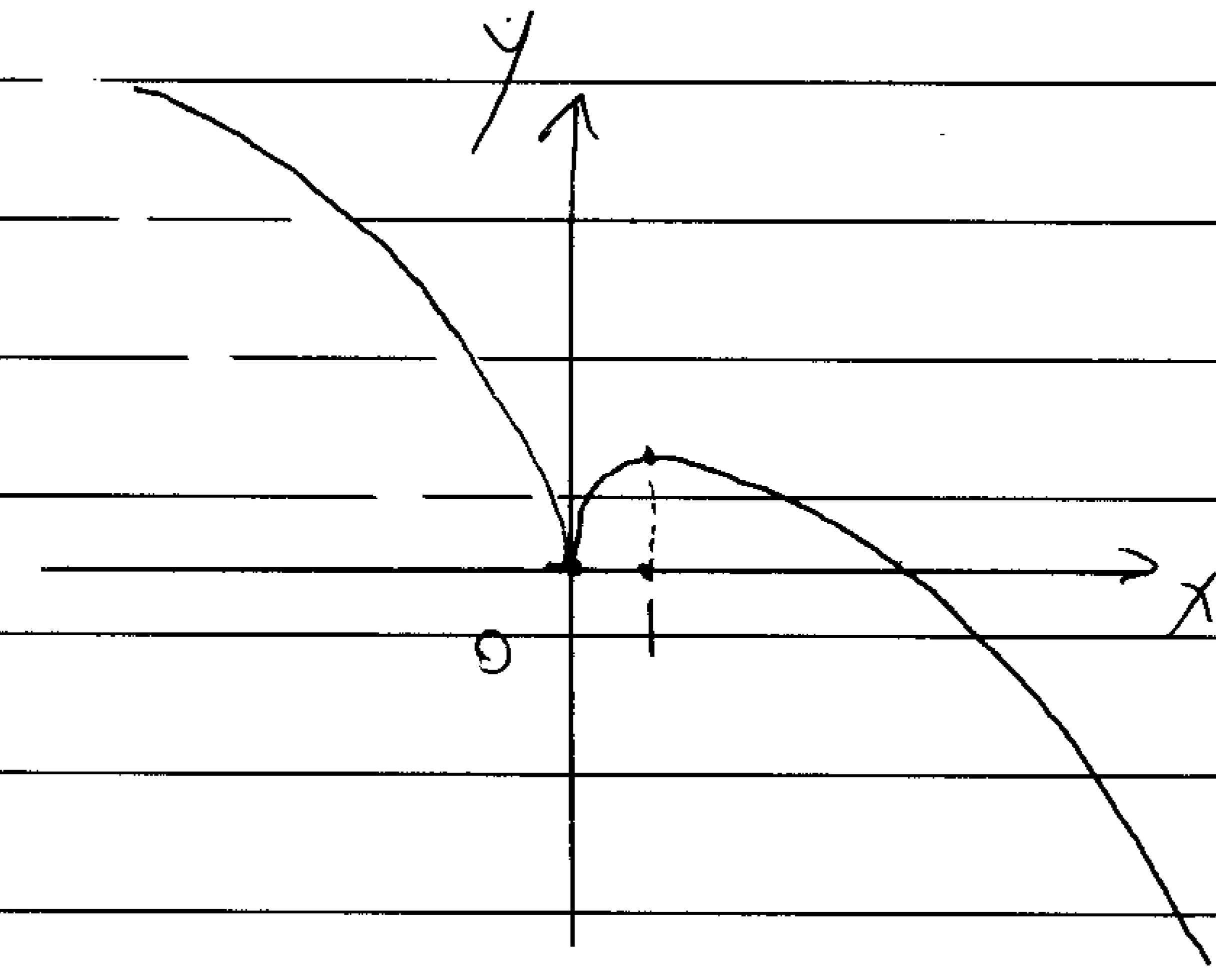
4.3-34



$y = 5x^{\frac{2}{5}} - 2x$

$\Rightarrow \frac{dy}{dx} = 2x^{-\frac{3}{5}} - 2$ & $\frac{d^2y}{dx^2} = -\frac{6}{5}x^{-\frac{8}{5}}$

$\frac{dy}{dx} = 0 \Rightarrow x = 1$ & $\left.\frac{d^2y}{dx^2}\right|_{x=1} = -\frac{6}{5} < 0$

$\Rightarrow$ y has local maximum at $x=1$

Moreover, y has no derivative at $x=0$

$\Rightarrow$

	$x<0$	$0<x<1$	$x>1$
	$\frac{dy}{dx} < 0$	$\frac{dy}{dx} > 0$	$\frac{dy}{dx} < 0$
(correction) $\Rightarrow$	$\frac{d^2y}{dx^2} < 0$	$\frac{d^2y}{dx^2} < 0$	$\frac{d^2y}{dx^2} < 0$

$\Rightarrow$ y has local minimum at $x=0$

75

$y = x^3 + bx^2 + cx + d$ have a point of inflection at $x=1$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=1} = 0$

$\frac{d^2y}{dx^2} = 6x + 2b \overset{x=1}{\Longrightarrow} 6 + 2b = 0 \Rightarrow b = -3$

78.

No!

$y = x^4$ has no inflection point at the origin,

even though $\frac{d^2y}{dx^2} = 0$ at $x=0$

80

$y = ax^3 + bx^2 + cx + d \;,\; a \neq 0$

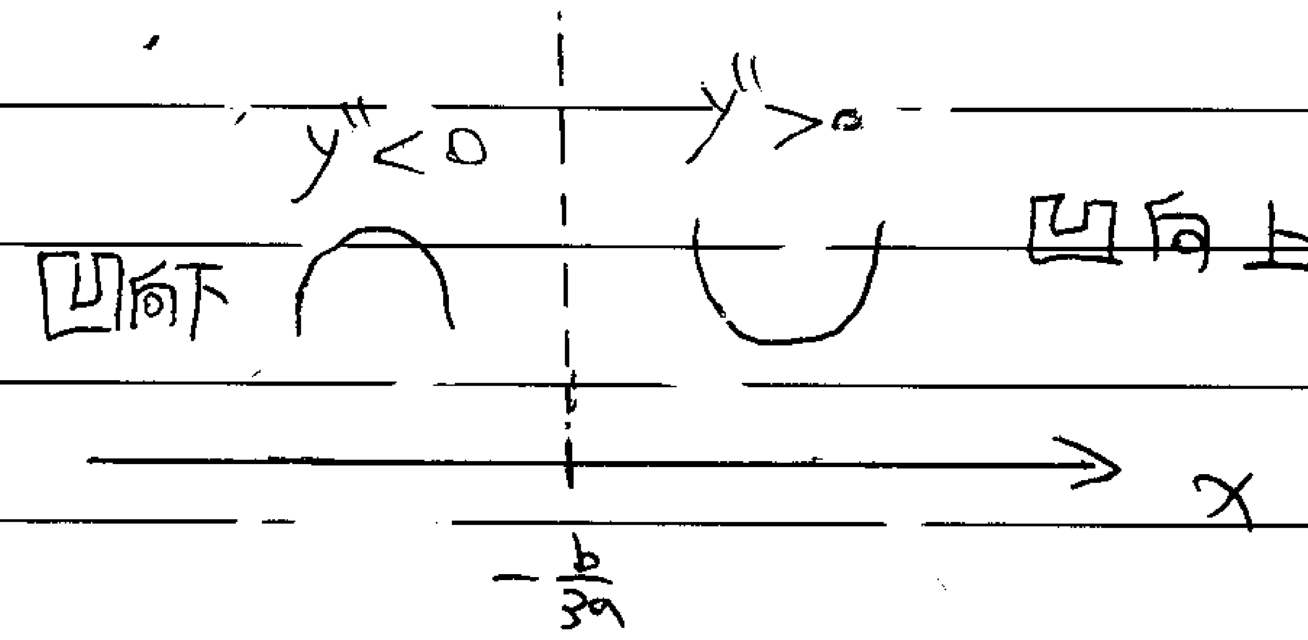
$\Rightarrow \frac{d^2y}{dx^2} = 6ax + 2b$

$\frac{d^2y}{dx^2} = 0 \Rightarrow x = -\frac{b}{3a}$

(i) If $a > 0 \Rightarrow \frac{d^2y}{dx^2} > 0 \;,\; x > -\frac{b}{3a}$

$\frac{d^2y}{dx^2} < 0 \;,\; x < -\frac{b}{3a}$

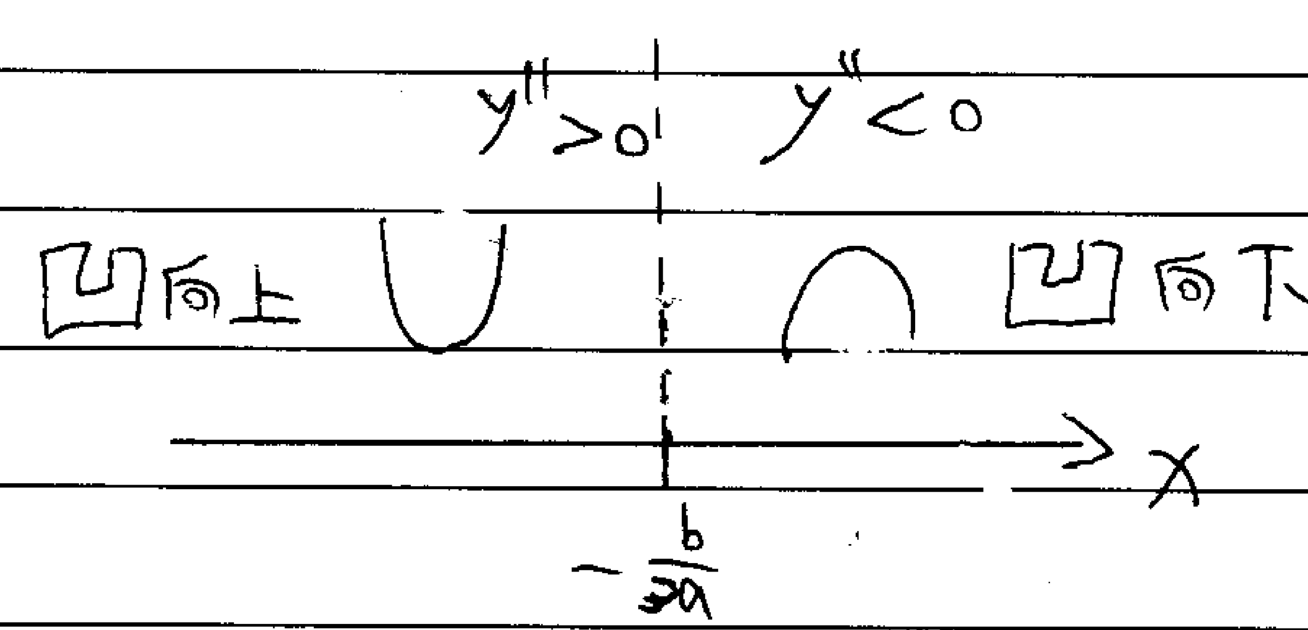
$\Rightarrow$



(ii) If $a < 0 \Rightarrow \frac{d^2y}{dx^2} < 0 \;,\; x > -\frac{b}{3a}$

$\frac{d^2y}{dx^2} > 0 \;,\; x < -\frac{b}{3a}$

$\Rightarrow$



Hw 07

4.4-41 $f(x)$ and $g(x)$ are polynomials in x and

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=2$$

$\Rightarrow$ If $g(x)=ax^m+(\text{degree}<m)$, then $f(x)=2ax^m+(\text{degree}<m)$

where $m\in\mathbb{N}\cup\{0\}$, $a\in\mathbb{R}\setminus\{0\}$

$$\Rightarrow \lim_{x\to-\infty}\frac{f(x)}{g(x)}=\lim_{x\to-\infty}\frac{2a+\frac{(\text{degree}<m)}{x^m}}{a+\frac{(\text{degree}<m)}{x^m}}=\frac{2a}{a}=2$$

58

Graph $y=x^{\frac{2}{3}}/(x^2-1)$

$\Rightarrow$ Vertical asymptotes : $x=1$, $x=-1$

$$y'=-\frac{2}{3}\frac{2x^2+1}{x^{\frac{1}{3}}(x^2-1)^2}$$

$\Rightarrow$ Critical points : $x=0, 1, -1$

$$y''=\frac{2}{9}\frac{14x^4+23x^2-1}{x^{\frac{4}{3}}(x^2-1)^3}=\frac{2}{9}\frac{\left(x^2-\frac{-23-\sqrt{585}}{28}\right)\left(x-\sqrt{\frac{-23+\sqrt{585}}{28}}\right)\left(x+\sqrt{\frac{-23+\sqrt{585}}{28}}\right)}{x^{\frac{4}{3}}(x^2-1)^3}$$

$\Rightarrow$ Inflection points: $x=\sqrt{\frac{-23+\sqrt{585}}{28}}, -\sqrt{\frac{-23+\sqrt{585}}{28}}$, where $\sqrt{\frac{-23+\sqrt{585}}{28}}\approx 0.206$

	①	②	③	④	⑤	⑥
$x^{\frac{1}{3}}$	−	−	−	+	+	+
y'	+	+	+	−	−	−
$\left(x^2-\frac{-23-\sqrt{585}}{28}\right)$	+	+	+	+	+	+
$\left(x^2-\frac{-23+\sqrt{585}}{28}\right)$	+	+	−	−	+	+
$x^{\frac{4}{3}}$	+	+	+	+	+	+
$(x^2-1)^3$	+	−	−	−	−	+
y''	+	−	+	+	−	+

① : $-2\le x<-1$

② : $-1<x<-\sqrt{\frac{-23+\sqrt{585}}{28}}$

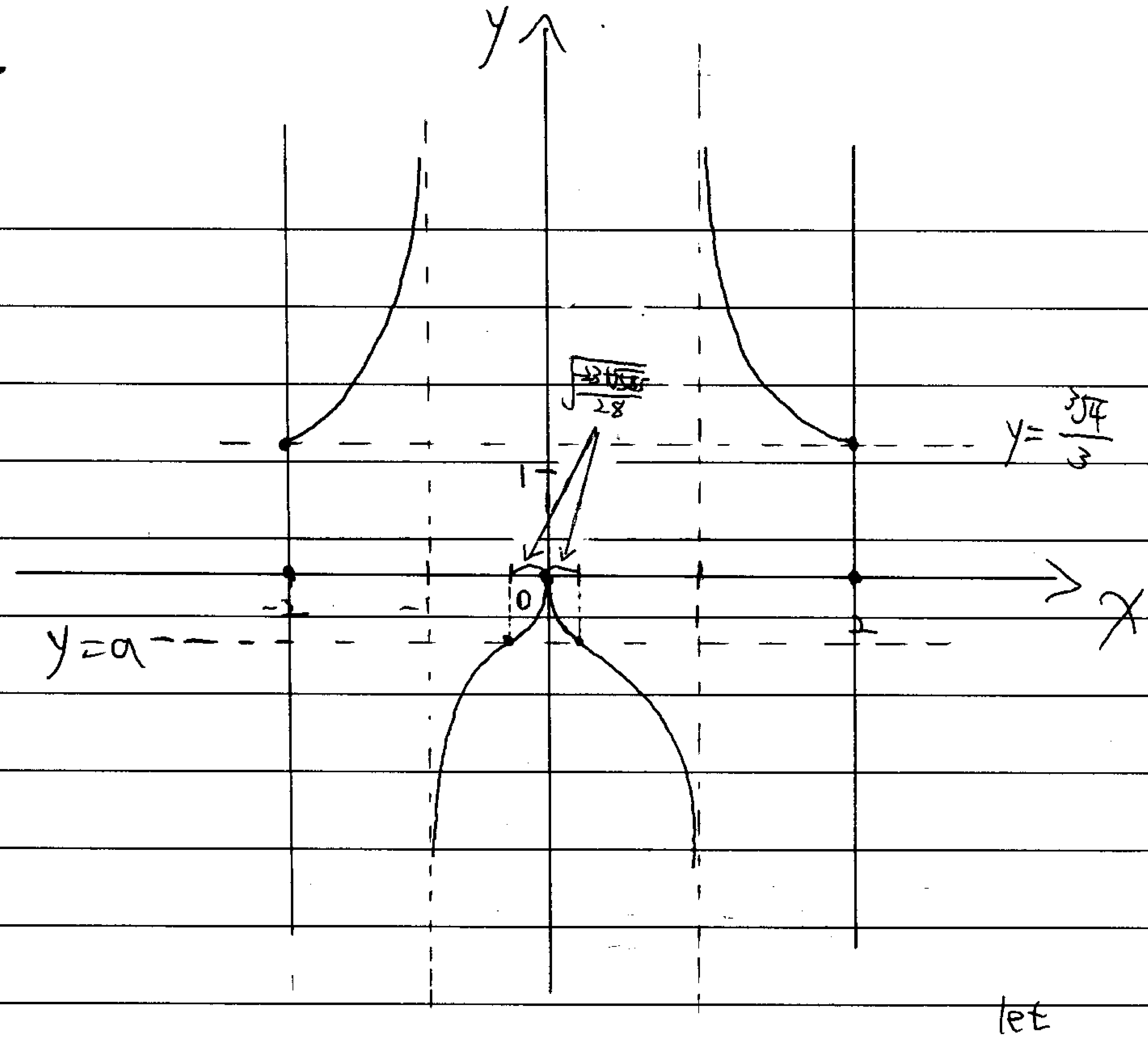
③ : $-\sqrt{\frac{-23+\sqrt{585}}{28}}<x<0$

④ : $0<x<\sqrt{\frac{-23+\sqrt{585}}{28}}$

⑤ : $\sqrt{\frac{-23+\sqrt{585}}{28}}<x<1$

⑥ : $1<x\le 2$

Per-Duet



$x=0,\ y=0 \Rightarrow (0,0)$, $x=-\sqrt{\frac{-23+\sqrt{585}}{28}}$, $y=\frac{x^{\frac{2}{3}}}{x^2-1} \overset{\text{let}}{=} a<0 \Rightarrow \left(-\sqrt{\frac{-23+\sqrt{585}}{28}}, a\right)$

$x=\sqrt{\frac{-23+\sqrt{585}}{28}}$, $y=a \Rightarrow \left(\sqrt{\frac{-23+\sqrt{585}}{28}}, a\right)$, $x=-2$, $y=\frac{\sqrt[3]{4}}{3} \Rightarrow \left(-2, \frac{\sqrt[3]{4}}{3}\right)$

$x=2$, $y=\frac{\sqrt[3]{4}}{3} \Rightarrow \left(2, \frac{\sqrt[3]{4}}{3}\right)$

4.5-26

$V(\text{volume}) = \pi x^2\cdot(y+3)/3$ and $x^2+y^2=9$

$$= \frac{\pi}{3}(9-y^2)(y+3)$$

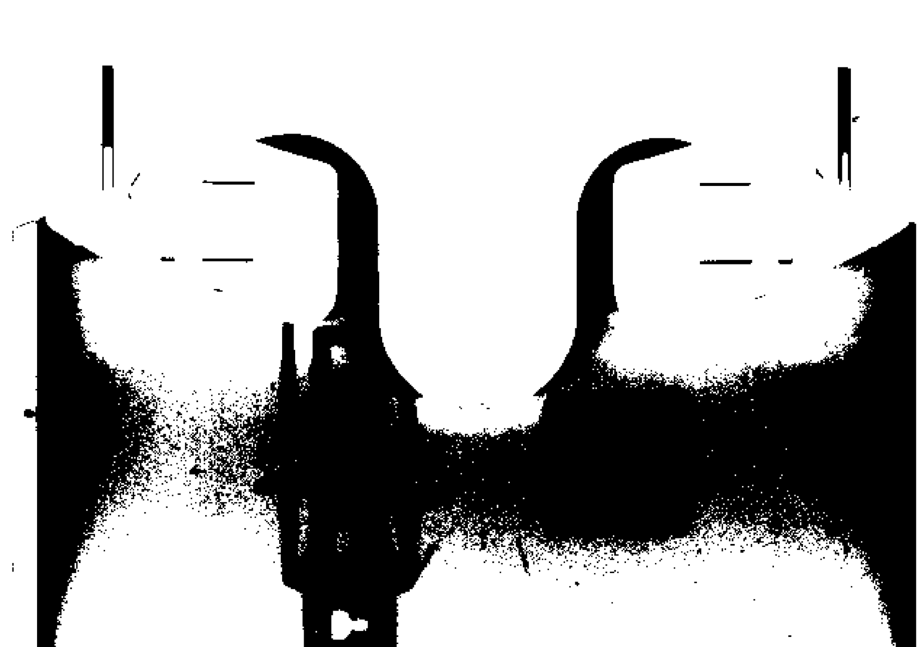
$$\Rightarrow \frac{dV}{dy} = \frac{\pi}{3}\left(-2y(y+3)+(9-y^2)\right) = \frac{\pi}{3}(-3y^2-6y+9) = -\pi(y^2+2y-3)$$

$$= -\pi(y-1)(y+3)$$

Hence when $y=1$, we have largest right circular cone

$(y-1)(y-(-3))$	+	−	+
$\frac{dV}{dy}$	−	+	−
		-3	1

$$\text{Volume} = \frac{\pi}{3}\cdot 8\cdot 4 = \frac{32}{3}\pi$$



36. $f(x)=3+4\cos x+\cos 2x$

(a) Since $\cos x$ has period 2π and $\cos 2x$ has period π

$\Rightarrow f$ has period 2π

(b)

$f(x)=3+4\cos x+\cos 2x$

$\Rightarrow f'(x)=-4\sin x-2\sin 2x=-4\sin x-4\sin x\cos x=-4\sin x(1+\cos x)$

If $0=f'(x) \Rightarrow \sin x(1+\cos x)=0$

$\Rightarrow \sin x=0$ or $\cos x=-1$

$\Rightarrow x=0, \pi, 2\pi$

$\sin x$	+	−	
$1+\cos x$	+	+	
f'	−	+	
	0	π	2π

Hence f has a local minimum

at $x=\pi \Rightarrow f(\pi)=3-4+1=0$

$\Rightarrow f\geq 0$