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$$f(x) = x^3 + 2x - 4, \quad f'(x) = 3x^2 + 2$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 + 2x_n - 4}{3x_n^2 + 2} = \frac{2x_n^3 + 4}{3x_n^2 + 2}$$

22.

Find a solution of the equation $\frac{1}{x^2+1} = x$

$$\text{Let } f(x) = x - \frac{1}{x^2+1}, \quad f'(x) = 1 + \frac{2x}{(x^2+1)^2}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n - \frac{1}{x_n^2+1}}{1 + \frac{2x_n}{(x_n^2+1)^2}} = \frac{3x_n^3 + 1}{(x_n^2+1)^2 + 2x_n}$$

4.2-31.

a) $f'(x) = (x-1)(x+2)(x-3)$

$f'(x) = 0 \Rightarrow x=1 \text{ or } x=-2 \text{ or } x=3$

$\Rightarrow x=1, -2, 3$ are critical points of f

b)

$x < -2$	$-2 < x < 1$	$1 < x < 3$	$x > 3$
$f'(x) < 0$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$

$\Rightarrow \begin{cases} f \text{ is increasing on } (-2, 1) \cup (3, \infty) \\ f \text{ is decreasing on } (-\infty, -2) \cup (1, 3) \end{cases}$

c)

According to the same table in b)

$\Rightarrow f$ have local minimum at $x = -2, 3$

& local maximum at $x = 1$

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a) $f'(x) = x^{-\frac{1}{3}}(x+2)$

$f'(x) = 0 \Rightarrow x = -2$, $f'(x)$ does not exist at $x = 0$

$\Rightarrow x = -2, 0$ are critical points of f

b)

$\begin{cases} f \text{ is increasing on } (-\infty, -2) \cup (0, \infty) \\ f \text{ is decreasing on } (-2, 0) \end{cases}$	$x < -2$	$-2 < x < 0$	$x > 0$
	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$

c) See text book's solution

57

$$g(x) = x^2 - 4x + 4, \quad 1 \leq x < \infty$$

$$\Rightarrow g'(x) = 2x - 4, \quad 1 \leq x < \infty$$

$$g'(x) = 0 \Rightarrow x = 2 \text{ (critical point)} \Rightarrow \begin{cases} g'(x) < 0, & x < 2 \\ g'(x) > 0, & x > 2 \end{cases}$$

$$\Rightarrow g(x) \text{ has local minimum } g(2) = 0 \text{ at } x = 2$$

$$\text{endpoints} \Rightarrow g(1) = 1 \text{ \& } g'(x) < 0, \quad 1 \leq x < 2$$

$$\Rightarrow g(x) \text{ has local maximum } 1 \text{ at } x = 1$$

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$$h(x) = \frac{x^3}{3} - 2x^2 + 4x, \quad 0 \leq x < \infty$$

$$\Rightarrow h'(x) = x^2 - 4x + 4 = (x-2)^2, \quad 0 \leq x < \infty$$

$$h'(x) = 0 \Rightarrow x = 2 \text{ (critical point)} \Rightarrow h'(x) \geq 0, \quad x \geq 0$$

$$\Rightarrow h(x) \text{ has no extreme value in } (0, \infty)$$

$$\text{endpoint} \Rightarrow h(0) = 0 \text{ \& } h'(x) > 0, \quad x \geq 0$$

$$\Rightarrow h(x) \text{ has local minimum } 0 \text{ at } x = 0$$

63 $y = x^3 - 6x$, $-2 \leq x \leq 2$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6 = 3(x^2 - 2)$, $-2 \leq x \leq 2$

$\frac{dy}{dx} = 0 \Rightarrow x = \sqrt{2}, -\sqrt{2}$ (critical points)

$\Rightarrow -2 < x < -\sqrt{2}$	$-\sqrt{2} < x < \sqrt{2}$	$\sqrt{2} < x < 2$
$\frac{dy}{dx} > 0$	$\frac{dy}{dx} < 0$	$\frac{dy}{dx} > 0$

$\Rightarrow y = x^3 - 6x$ has local maximum $4\sqrt{2}$ at $x = -\sqrt{2}$

& local minimum $-4\sqrt{2}$ at $x = \sqrt{2}$

endpoints: $y(-2) = 4$ & $\frac{dy}{dx} > 0$, $-2 < x < -\sqrt{2}$

$\Rightarrow y$ has local minimum 4 at $x = -2$

$y(2) = -4$ & $\frac{dy}{dx} > 0$, $\sqrt{2} < x < 2$

$\Rightarrow y$ has local maximum -4 at $x = 2$

81

a) $g(x) = \frac{1}{x} \Rightarrow g'(x) = -\frac{1}{x^2} < 0$, $x \neq 0$

$\Rightarrow g$ is decreasing at $x \neq 0$

b)

g is decreasing on $(-\infty, 0) \cup (0, \infty)$

but g is not continuous at $x = 0$

