

3.6-58

$$\frac{ds}{dt} = v = k\sqrt{s}$$

acceleration (加速度) is denoted by "a"

$$a = \frac{d^2s}{dt^2}$$

$$= \frac{d}{dt} \left(\frac{ds}{dt} \right) = \frac{d}{dt} (k\sqrt{s}) = k \frac{1}{2\sqrt{s}} \cdot \frac{ds}{dt} \quad \Leftarrow \text{chain rule}$$

$$= \frac{k}{2\sqrt{s}} \cdot k\sqrt{s} = \frac{k^2}{2}$$

$\Rightarrow$ acceleration is a constant

3.1-10 $f(x) = 1+x$, $x_0 = 8.1$

$$\Rightarrow f(x) \approx L(x) = f(8) + f'(8)(x-8) = 9 + 1 \cdot (x-8) = x+1$$

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$$(1+x)^k \approx 1+kx$$

a) $(1.0002)^{50} \approx 1 + 50 \cdot 0.0002 = 1 + 0.01 = 1.01$

b) $\sqrt[3]{1.009} = (1+0.009)^{\frac{1}{3}} \approx 1 + \frac{1}{3} \cdot 0.009 = 1.003$

Use $f(x) - L(x) = \frac{1}{2} f''(c)(x-x_0)^2$ & the error bound in Problem 3

$$\Rightarrow \sqrt[3]{1.009} - 1.003 = \frac{1}{2} \left[\frac{1}{3} \cdot \left(-\frac{2}{3}\right) (1+c)^{-\frac{5}{3}} \right] (0.009-0)^2 \text{ where } c \text{ lies between } 0 \text{ \& } 0.009$$

$$\Rightarrow |\sqrt[3]{1.009} - 1.003| \leq \frac{1}{9} \cdot (1+0)^{-\frac{5}{3}} \cdot 0.009^2 = 9 \times 10^{-6}$$

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W : weight, r : radius, m : density (constant), h : high

$$W = \pi r^2 h \cdot m \Rightarrow dW = 2\pi r h m dr \Rightarrow \frac{dW}{W} = \frac{2\pi r h m dr}{\pi r^2 h m} = 2 \frac{dr}{r}$$

$$\frac{dW}{W} = \frac{1}{1000} = 0.1\% \Rightarrow \frac{dr}{r} = \frac{1}{2} \frac{dW}{W} = 0.05\%$$

51. $f(x)$ has a horizontal tangent at $x=a \Rightarrow f'(a)=0$

$$\Rightarrow L(x) = f(a) + f'(a)(x-a) = f(a) : \text{constant function}$$

Ch 3-90 $f(x) = \frac{1}{1+\tan x} \Rightarrow f'(x) = -\frac{\sec^2 x}{(1+\tan x)^2}$

$$\Rightarrow L(x) = f(0) + f'(0)(x-0) = 1 + (-1)x = 1-x$$

"For $\frac{1}{1+\sin(2x)}$ "

$$\frac{1}{1+x} \approx 1-x \quad \& \quad \sin 2x \approx 0 + 2 \cdot (x-0) = 2x$$

$$\Rightarrow \frac{1}{1+\sin(2x)} \approx 1-2x$$

Check! $g(x) = \frac{1}{1+\sin 2x} \Rightarrow g'(x) = -\frac{2 \cos 2x}{(1+\sin 2x)^2}$

$$\Rightarrow L(x) = 1 + (-2) \cdot (x-0) = 1-2x$$