3.1-40

(a) differentiable: $x \in [-2,3] \setminus \{-1,0,2\}$

(b) continuous but not differentiable: $x=-1$

(c) neither continuous nor differentiable: $x=0$ & $x=2$

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Suppose that $g(t)$ and $h(t)$ are defined for all values of t and

$g(0)=h(0)=0$, Can $\lim_{t\to 0} \frac{g(t)}{h(t)}$ exist? If it does exist, must it be 0?

Let $g(t)=2t$, $h(t)=t$

$\Rightarrow \lim_{t\to 0} \frac{g(t)}{h(t)} = 2$

So the limit may exist and may not be 0

3.2-57

(a) $y = f(x) = x^3 - 4x + 1 \Rightarrow f'(x) = 3x^2 - 4$

$f'(2) = 8 \Rightarrow$ Slope of the equation perpendicular to the tangent to the curve : $-\frac{1}{8}$

Equation : $y = -\frac{1}{8}(x-2) + 1 = -\frac{1}{8}x + \frac{5}{4}$

(b)

$f''(x) = 6x$, solve $6x = 0 \Rightarrow x = 0$

Moreover , $\underbrace{f''(x) < 0 \text{ , } x < 0}_{\text{decreasing}}$ & $\underbrace{f''(x) > 0 \text{ , } x > 0}_{\text{increasing}}$

Hence we have the smallest slope $f'(0) = -4$ at $(0, 1)$.

3.2-57 (c) Solve $f'(x) = 3x^2 - 4 = 8$

$\Rightarrow 3x^2 = 12 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

$f(2) = 1 \Rightarrow y = 8(x-2)+1 = 8x - 15$

$f(-2) = 1 \Rightarrow y = 8(x+2)+1 = 8x+17$

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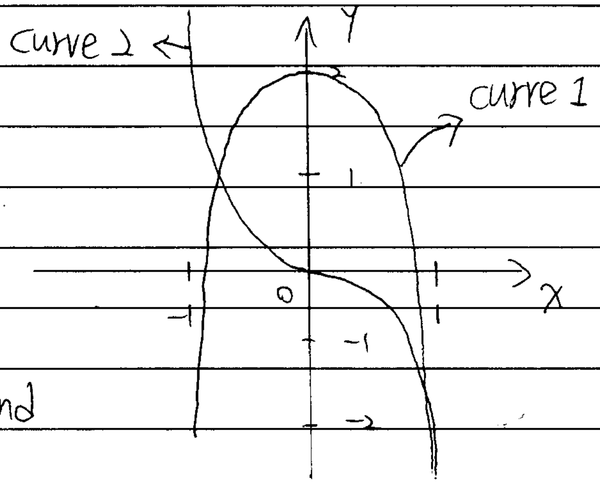

$f(x)$ is curve 1
$f'(x)$ is curve 2

Since all the slope of the tangent line to curve 2 is negative, and there are positive values in curve 1.

So curve 1 can not be the derivative of curve 2

Consider the slope of curve 1, we can find out that the slope decreasing on $(-1, 1)$ and have 0 derivative at $x=0$, curve 2 satisfy all the phenomenon we found in curve 1

§3.5-extra1 $\frac{d}{dx}\left(f_1(x) f_2(x) \cdots f_n(x)\right) = f_1'(x) \cdot f_2(x) \cdots f_n(x) + f_1(x) \cdot \left(f_2(x) \cdots f_n(x)\right)'$

$= f_1'(x) \cdot f_2(x) \cdots f_n(x) + f_1(x) \cdot f_2'(x) \cdot f_3(x) \cdots f_n(x) + f_1(x) \cdot f_2(x) \cdot \left(f_3(x) \cdots f_n(x)\right)'$

$= \cdots$

$= f_1'(x) f_2(x) \cdots f_n(x) + f_1(x) \cdot f_2'(x) \cdot f_3(x) \cdots f_n(x) + \cdots$

$+ \underbrace{f_1(x) \cdots f_{i-1}(x) \cdot f_i'(x) f_{i+1}(x) \cdots f_n(x)}_{\text{the ith term}} + \cdots + f_1(x) \cdots f_{n-1}(x) \cdot f_n'(x)$

$\frac{d^n}{dx^n}\left(f(x) g(x)\right) = \sum_{k=0}^{n} C_k^n \left(\frac{d^k f}{dx^k}\right)\left(\frac{d^{n-k} g}{dx^{n-k}}\right)$

§3.5-extra 2

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix} = \frac{d}{dx}\left(f(x)k(x) - g(x)h(x)\right)$$

$$= f'(x)k(x) + f(x)k'(x) - \left(g'(x)h(x) + g(x)h'(x)\right)$$

$$= \left(f'(x)k(x) - g(x)h'(x)\right) + \left(f(x)k'(x) - g'(x)h(x)\right)$$

$$= \begin{vmatrix} f'(x) & g(x) \\ h'(x) & k(x) \end{vmatrix} + \begin{vmatrix} f(x) & g'(x) \\ h(x) & k'(x) \end{vmatrix}$$

The other equality is similar

Per-Duet

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & n(x) \end{vmatrix} = \frac{d}{dx}\left(f(x)\begin{vmatrix} j(x) & k(x) \\ m(x) & n(x) \end{vmatrix} - i(x)\begin{vmatrix} g(x) & h(x) \\ m(x) & n(x) \end{vmatrix} + l(x)\begin{vmatrix} g(x) & h(x) \\ j(x) & k(x) \end{vmatrix} \right)$$

$$= \underbrace{f'(x)\begin{vmatrix} j(x) & k(x) \\ m(x) & n(x) \end{vmatrix}}_{\textcircled{1}} + f(x)\left(\underbrace{\begin{vmatrix} j'(x) & k(x) \\ m'(x) & n(x) \end{vmatrix}}_{\textcircled{2}} + \underbrace{\begin{vmatrix} j(x) & k'(x) \\ m(x) & n'(x) \end{vmatrix}}_{\textcircled{3}} \right) - \underbrace{i'(x)\begin{vmatrix} g(x) & h(x) \\ m(x) & n(x) \end{vmatrix}}_{\textcircled{1}}$$

$$- i(x)\left(\underbrace{\begin{vmatrix} g'(x) & h(x) \\ m'(x) & n(x) \end{vmatrix}}_{\textcircled{2}} + \underbrace{\begin{vmatrix} g(x) & h'(x) \\ m(x) & n'(x) \end{vmatrix}}_{\textcircled{3}} \right) + \underbrace{l'(x)\begin{vmatrix} g(x) & h(x) \\ j(x) & k(x) \end{vmatrix}}_{\textcircled{1}} + l(x)\left(\underbrace{\begin{vmatrix} g'(x) & h(x) \\ j'(x) & k(x) \end{vmatrix}}_{\textcircled{2}} + \underbrace{\begin{vmatrix} g(x) & h'(x) \\ j(x) & k'(x) \end{vmatrix}}_{\textcircled{3}} \right)$$

$$= \underbrace{\begin{vmatrix} f'(x) & g(x) & h(x) \\ i'(x) & j(x) & k(x) \\ l'(x) & m(x) & n(x) \end{vmatrix}}_{\textcircled{1}} + \underbrace{\begin{vmatrix} f(x) & g'(x) & h(x) \\ i(x) & j'(x) & k(x) \\ l(x) & m'(x) & h(x) \end{vmatrix}}_{\textcircled{2}} + \underbrace{\begin{vmatrix} f(x) & g(x) & h'(x) \\ i(x) & j(x) & k'(x) \\ l(x) & m(x) & n'(x) \end{vmatrix}}_{\textcircled{3}}$$

Similarly, we have

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & n(x) \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & h(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ i'(x) & j'(x) & k'(x) \\ l(x) & m(x) & n(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l'(x) & m'(x) & n'(x) \end{vmatrix}$$

And the 4X4

$$\frac{d}{dx}\begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} = \begin{vmatrix} a'(x) & b'(x) & c'(x) & d'(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} + \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e'(x) & f'(x) & g'(x) & h'(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix}$$

$$+ \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i'(x) & j'(x) & k'(x) & l'(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} + \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m'(x) & n'(x) & p'(x) & q'(x) \end{vmatrix}$$