HW01

2.2 − 72(b)

$$\lim_{x\to\infty}\left(\sqrt{x+54}-\sqrt{x}\right)=\lim_{x\to\infty}\left(\sqrt{x+54}-\sqrt{x}\right)\cdot\frac{\sqrt{x+54}+\sqrt{x}}{\sqrt{x+54}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{54}{\sqrt{x+54}+\sqrt{x}}=0$$

2.3 − 40

Since $\frac{x-1}{x}<\frac{\lfloor x\rfloor}{x}\le 1$, $x\neq 0$

and $\lim_{x\to\infty}\frac{x-1}{x}=1 \Rightarrow \lim_{x\to\infty}\frac{\lfloor x\rfloor}{x}=1$

(By Sandwich Theorem)

$\lim_{x\to-\infty}\frac{x-1}{x}=1 \Rightarrow \lim_{x\to-\infty}\frac{\lfloor x\rfloor}{x}=1$

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(a) We can't conclude anything about the value of f, g and h at $x=2$

Because limitation only describe the behavior around $x=2$, not exactly the behavior at $x=2$.

(b)

$f(2)=0$ is possible.

Ex: $f(x)=\begin{cases}2(x-2)-5, & x\neq 2\\ 0, & x=2\end{cases}$, $g(x)=\begin{cases}3(x-2)-5, & x<2\\ -1, & x=2\\ (x-2)-5, & x>2\end{cases}$

$h(x)=\begin{cases}(x-2)-5, & x<2\\ 1, & x=2\\ 3(x-2)-5, & x>2\end{cases}$

(c) $\lim_{x\to 2}f(x)$ can't be 0, or it will make a contradiction to the Sandwich theorem.

Per-Duet

2.4-54

Let $g(x) = \begin{cases} -x, & x < -1 \\ -x-2, & x \geq -1 \end{cases}$

Since $\lim_{x \to -1^-} g(x) = 1$ but $\lim_{x \to -1^+} g(x) = -1$

$\Rightarrow$ $g(x)$ has a nonremovable discontinuity.

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It's possible!

Let $f(x) = 1$ & $g(x) = x - \frac{1}{2}$ are both continuous on $[0, 1]$

$\Rightarrow \frac{f(x)}{g(x)} = \frac{1}{x - \frac{1}{2}}$ & $\lim_{x \to \frac{1}{2}^+} \frac{f(x)}{g(x)} = \infty$, $\lim_{x \to \frac{1}{2}^-} \frac{f(x)}{g(x)} = -\infty$

$\frac{f(x)}{g(x)}$ is discontinuous at $x = \frac{1}{2}$

56.

Let $f(x) = \begin{cases} 2, & x = 0 \\ 1, & x \neq 0 \end{cases}$ & $g(x) = \begin{cases} \frac{1}{2}, & x = 0 \\ 1, & x \neq 0 \end{cases}$

Both of f & g are discontinuous at $x = 0$

But $h(x) = f(x) \cdot g(x) = 1$ everywhere $\Rightarrow h(x) = 1$ is continuous at $x = 0$

57. Yes, it is true.

Since if $f(x)$ is continuous & $f(x)$ changes sign

$\Rightarrow \exists\, a, b$ s.t. $f(a)>0$ & $f(b)<0$

By I.V.T., we can find $c \in [a,b]$ s.t. $f(c)=0$ $(\rightarrow\leftarrow)$

Chap 2-42

$\lim_{x\to 0^+} \frac{[\sin x]}{x} = \lim_{x\to 0^+} \frac{0}{x}$, as $x \in (0, \frac{\pi}{2})$

$= 0$

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Let $f(x) = \cos x - 2 + x^3$ is continuous everywhere

Since $f(2) = \cos 2 - 2 + 8 = \cos 2 + 6 > 0$

$f(0) = \cos 0 - 2 + 0 = 1 - 2 = -1 < 0$

By I.V.T., we can find $c \in [0,2]$ s.t. $f(c)=0$

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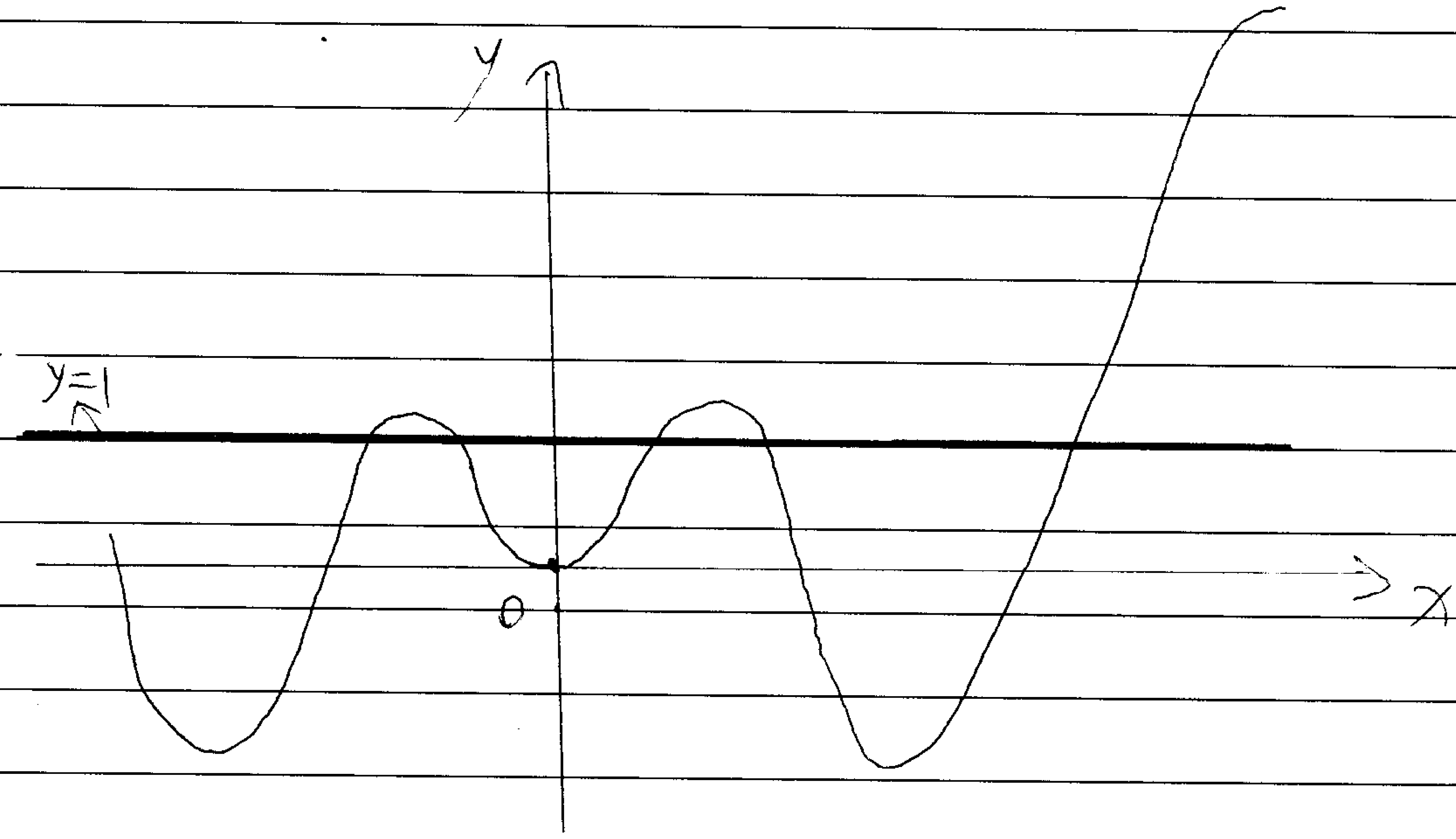
Let $g(x) = x\sin x - 1$ is continuous everywhere

Since $g(0) = 0 - 1 = -1 < 0$ & $g(2) = 2\cdot\sin 2 - 1 > 2\cdot\frac{1}{2} - 1 = 0$

By I.V.T., there's a $c \in [0,2]$ s.t. $g(c)=0$

Per-Duet

There are infinitely many solutions of the equation $x\sin x = 1$



When $x \to \infty$ or $x \to -\infty$

$y=1$ & $y = x\sin x$ would cross with each other infinitely many times.

S2.3-extra | (a) "$x \to c^+$"

$\Rightarrow$ for all x in the interval $(c, c+\delta)$ for some $\delta > 0$

(b) "$x \to \infty$"

$\Rightarrow$ for all x in the interval $[M, \infty)$ for some $M \in \mathbb{R}$

S2.3-extra)

$$\frac{\sin\theta-\frac{1}{2}}{\theta-\frac{\pi}{6}}=\frac{\sin\theta-\sin\frac{\pi}{6}}{\theta-\frac{\pi}{6}}=\frac{2\cos\left(\frac{\theta+\frac{\pi}{6}}{2}\right)\sin\left(\frac{\theta-\frac{\pi}{6}}{2}\right)}{\theta-\frac{\pi}{6}}$$

$$=\cos\left(\frac{\theta+\frac{\pi}{6}}{2}\right)\cdot\frac{\sin\left(\frac{\theta-\frac{\pi}{6}}{2}\right)}{\left(\frac{\theta-\frac{\pi}{6}}{2}\right)}$$

$$\Rightarrow \lim_{\theta\to\frac{\pi}{6}}\frac{\sin\theta-\frac{1}{2}}{\theta-\frac{\pi}{6}}=\frac{\sqrt{3}}{2}\cdot 1=\frac{\sqrt{3}}{2}$$