25–30 $f(x)=\sqrt{1-5x}$, $x_0=-3$, $\epsilon=0.5$

$\lim_{x\to -3}\sqrt{1-5x} = 4$

$\left|\sqrt{1-5x}-4\right| < \epsilon = 0.5 \iff -\frac{1}{2} < \sqrt{1-5x}-4 < \frac{1}{2}$

$\iff \frac{7}{2} < \sqrt{1-5x} < \frac{9}{2}$

$\iff \frac{49}{4} < 1-5x < \frac{81}{4}$

$\iff \frac{45}{4} < -5x < \frac{77}{4} \iff -\frac{77}{20} < x < -\frac{9}{4}$

$\iff -\frac{17}{20} < x-(-3) < \frac{3}{4} = \frac{15}{20}$

Take $\delta = \min\left\{\left|\frac{-17}{20}\right|, \left|\frac{15}{20}\right|\right\} = \frac{15}{20} = \frac{3}{4}$

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$f(x)=\frac{4}{x}$, $x_0=\frac{1}{2}$, $\epsilon=0.04$

$\lim_{x\to \frac{1}{2}} \frac{4}{x} = 8$

$\left|\frac{4}{x}-8\right| < \epsilon = 0.04 \iff -0.04 < \frac{4}{x}-8 < 0.04 \iff 7.96 < \frac{4}{x} < 8.04$

$\iff \frac{1}{2.01} < x < \frac{1}{1.99} \iff \frac{1}{2.01}-\frac{1}{2} < x-\frac{1}{2} < \frac{1}{1.99}-\frac{1}{2}$

Take $\delta = \min\left\{\left|\frac{1}{2.01}-\frac{1}{2}\right|, \left|\frac{1}{1.99}-\frac{1}{2}\right|\right\}$

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38.

$$f(x)=\frac{|x-5|}{x-5}=\begin{cases}1, & x>5\\ -1, & x<5\end{cases}$$

(a) $\epsilon=4$, $-3<f(x)<5 \Rightarrow x\in\mathbb{R}\setminus\{5\}$

(b) $\epsilon=2$, $-1<f(x)<3 \Rightarrow x>5$

(c) $\epsilon=1$, $0<f(x)<2 \Rightarrow x>5$

(d) $\epsilon=\frac{1}{2}$, $\frac{1}{2}<f(x)<\frac{3}{2} \Rightarrow x>5$

41.

Let $f(x)=\begin{cases}\sin x, & x\neq\frac{\pi}{2}\\ 2, & x=\frac{\pi}{2}\end{cases}$

Consider the point $x=\frac{\pi}{2}$, $f(x)$ increases to 1 as $x\to\frac{\pi}{2}^{+}$ or $x\to\frac{\pi}{2}^{-}$

So $f(x)$ gets closer to 2, but $\lim_{x\to\frac{\pi}{2}} f(x)=1$

Since given $\epsilon>0$, $\forall\,\delta>0$, $\exists\, x\in(\frac{\pi}{2}-\delta, \frac{\pi}{2}+\delta)$

such that $|f(x)-2|>1$

(No matter how closer we take on the domain, there's always a distance between $\lim_{x\to\frac{\pi}{2}} f(x)$ & $f(\frac{\pi}{2})$)

Chap 2-74 $\lim_{x \to 0} |x| = 0$

$\big||x| - 0\big| < \epsilon \Rightarrow |x| < \epsilon \Rightarrow |x - 0| < \epsilon$

So, given $\epsilon > 0$, we take $\delta = \epsilon$

then for all x, if $|x - 0| < \delta \Rightarrow |f(x) - 0| = \big||x| - 0\big| = |x| < \delta = \epsilon$

75 $f(x) = 5x - 10$, $x_0 = 3$, $\epsilon = 0.05$

$\lim_{x \to 3} 5x - 10 = 5$

$|5x - 10 - 5| < 0.05 \Leftrightarrow -0.05 < 5x - 15 < 0.05$

$\Leftrightarrow 14.95 < 5x < 15.05$

$\Leftrightarrow 2.99 < x < 3.01 \Leftrightarrow -0.01 < x - 3 < 0.01$

We take $\delta = 0.01$

Problem 3.

(a) $\lim_{x \to c^+} f(x) = L$: Given $\epsilon > 0$, $\exists \delta > 0$, s.t. for all x, $c < x < c + \delta$

$\Rightarrow |f(x) - L| < \epsilon$

(b) $\lim_{x \to c} f(x) = \infty$: Given $M \in \mathbb{R}$, $\exists \delta > 0$, s.t. for all x, $0 < |x - c| < \delta$

$\Rightarrow f(x) > M$

(c) $\lim_{x \to -\infty} f(x) = L$: Given $\epsilon > 0$, $\exists M \in \mathbb{R}$ s.t. for all x, $x < M$

$\Rightarrow |f(x) - L| < \epsilon$

4. If $f(x)$ and $g(x)$ are both continuous at $x=c$

$\Rightarrow$ Given $\epsilon>0$, $\exists\, \delta_1>0$ s.t. for all x, $0<|x-c|<\delta_1 \Rightarrow |f(x)-f(c)|<\frac{\epsilon}{2}$

$\exists\, \delta_2>0$ s.t. for all x, $0<|x-c|<\delta_2 \Rightarrow |g(x)-g(c)|<\frac{\epsilon}{2}$

take $\delta=\min\{\delta_1,\delta_2\}$, then for all x, $0<|x-c|<\delta$

$$\Rightarrow |f(x)+g(x)-(f(c)+g(c))| = |(f(x)-f(c))+(g(x)-g(c))|$$

$$\leq |f(x)-f(c)|+|g(x)-g(c)| < \epsilon$$

$$\& \ |f(x)-g(x)-(f(c)-g(c))| = |(f(x)-f(c))-(g(x)-g(c))|$$

$$\leq |f(x)-f(c)|+|g(x)-g(c)| < \epsilon$$

Hence both $f(x)+g(x)$ & $f(x)-g(x)$ are continuous at $x=c$

5 $f(x)$ is continuous at $x=c$

$\Rightarrow$ Given $\epsilon>0$, $\exists \tilde{\delta}>0$ s.t. for all x, $0<|x-c|<\tilde{\delta} \Rightarrow |f(x)-f(c)|<\frac{\epsilon}{3}$

take $\delta=\tilde{\delta}$ $\Rightarrow |3f(x)-3f(c)| = 3|f(x)-f(c)| < 3\cdot\frac{\epsilon}{3} = \epsilon$

Hence $3f(x)$ is continuous at $x=c$

3.1-40

(a) differentiable: $x\in[-2,3]\setminus\{-1,0,2\}$

(b) continuous but not differentiable: $x=-1$

(c) neither continuous nor differentiable: $x=0$ & $x=2$

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Suppose that $g(t)$ and $h(t)$ are defined for all values of t and $g(0)=h(0)=0$, Can $\lim_{t\to0}\frac{g(t)}{h(t)}$ exist? If it does exist, must it be 0?

Let $g(t)=2t$, $h(t)=t$

$\Rightarrow \lim_{t\to0}\frac{g(t)}{h(t)}=2$

So the limit may exist and may not be 0

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