2.4 − 54

Let $g(x) = \begin{cases} -x, & x < -1 \\ -x-2, & x \geq -1 \end{cases}$

Since $\lim\limits_{x \to -1^-} g(x) = 1$ but $\lim\limits_{x \to -1^+} g(x) = -1$

$\Rightarrow$ $g(x)$ has a nonremovable discontinuity.

55

It's possible!

Let $f(x) = 1$ & $g(x) = x - \frac{1}{2}$ are both continuous on $[0, 1]$

$\Rightarrow \frac{f(x)}{g(x)} = \frac{1}{x - \frac{1}{2}}$ & $\lim\limits_{x \to \frac{1}{2}^+} \frac{f(x)}{g(x)} = \infty$, $\lim\limits_{x \to \frac{1}{2}^-} \frac{f(x)}{g(x)} = -\infty$

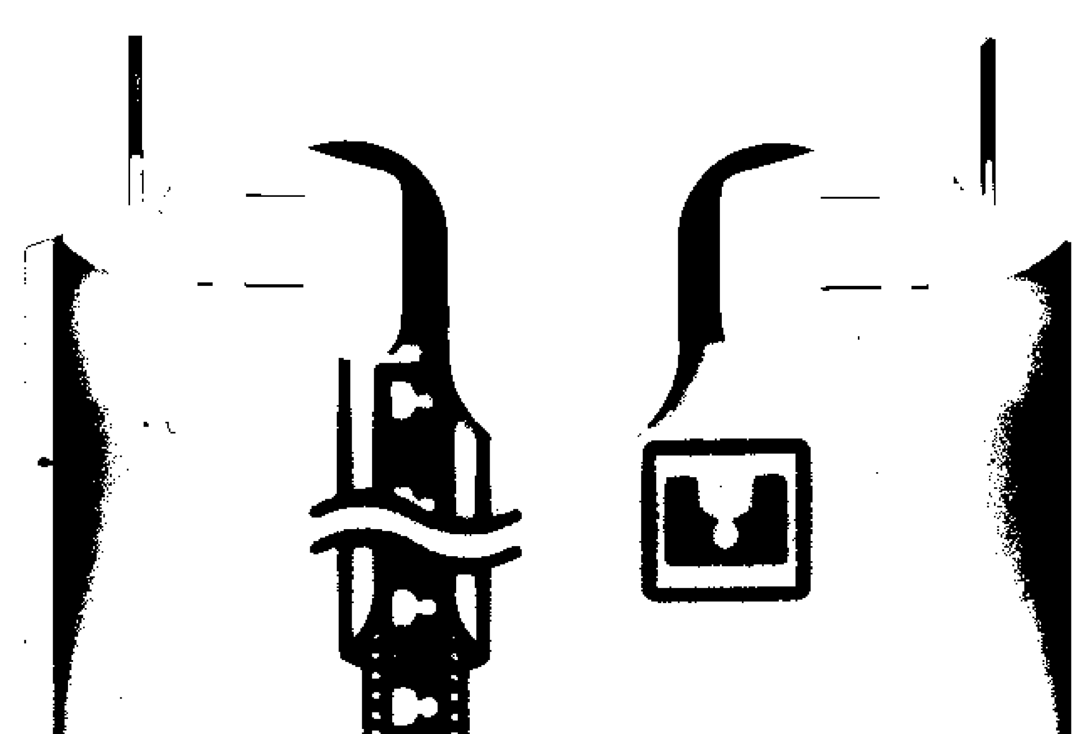
$\frac{f(x)}{g(x)}$ is discontinuous at $x = \frac{1}{2}$

56.

Let $f(x) = \begin{cases} 2, & x = 0 \\ 1, & x \neq 0 \end{cases}$ & $g(x) = \begin{cases} \frac{1}{2}, & x = 0 \\ 1, & x \neq 0 \end{cases}$

Both of $f$ & $g$ are discontinuous at $x = 0$

But $h(x) = f(x) \cdot g(x) = 1$ everywhere $\Rightarrow h(x) = 1$ is continuous at $x = 0$



57. Yes, it is true.

Since if $f(x)$ is a continuous & $f(x)$ changes sign

$\Rightarrow \exists\ a, b$ s.t. $f(a) > 0$ & $f(b) < 0$

By I.V.T., we can find $c \in [a, b]$ s.t. $f(c) = 0$ $(\rightarrow\leftarrow)$

Chap 2-42

$\lim_{x \to 0^+} \frac{\lfloor \sin x \rfloor}{x} = \lim_{x \to 0^+} \frac{0}{x}$, as $x \in (0, \frac{\pi}{2})$

$= 0$

69

Let $f(x) = \cos x - 2 + x^3$ is continuous everywhere

Since $f(2) = \cos 2 - 2 + 8 = \cos 2 + 6 > 0$

$f(0) = \cos 0 - 2 + 0 = 1 - 2 = -1 < 0$

By I.V.T., we can find $c \in [0, 2]$ s.t. $f(c) = 0$

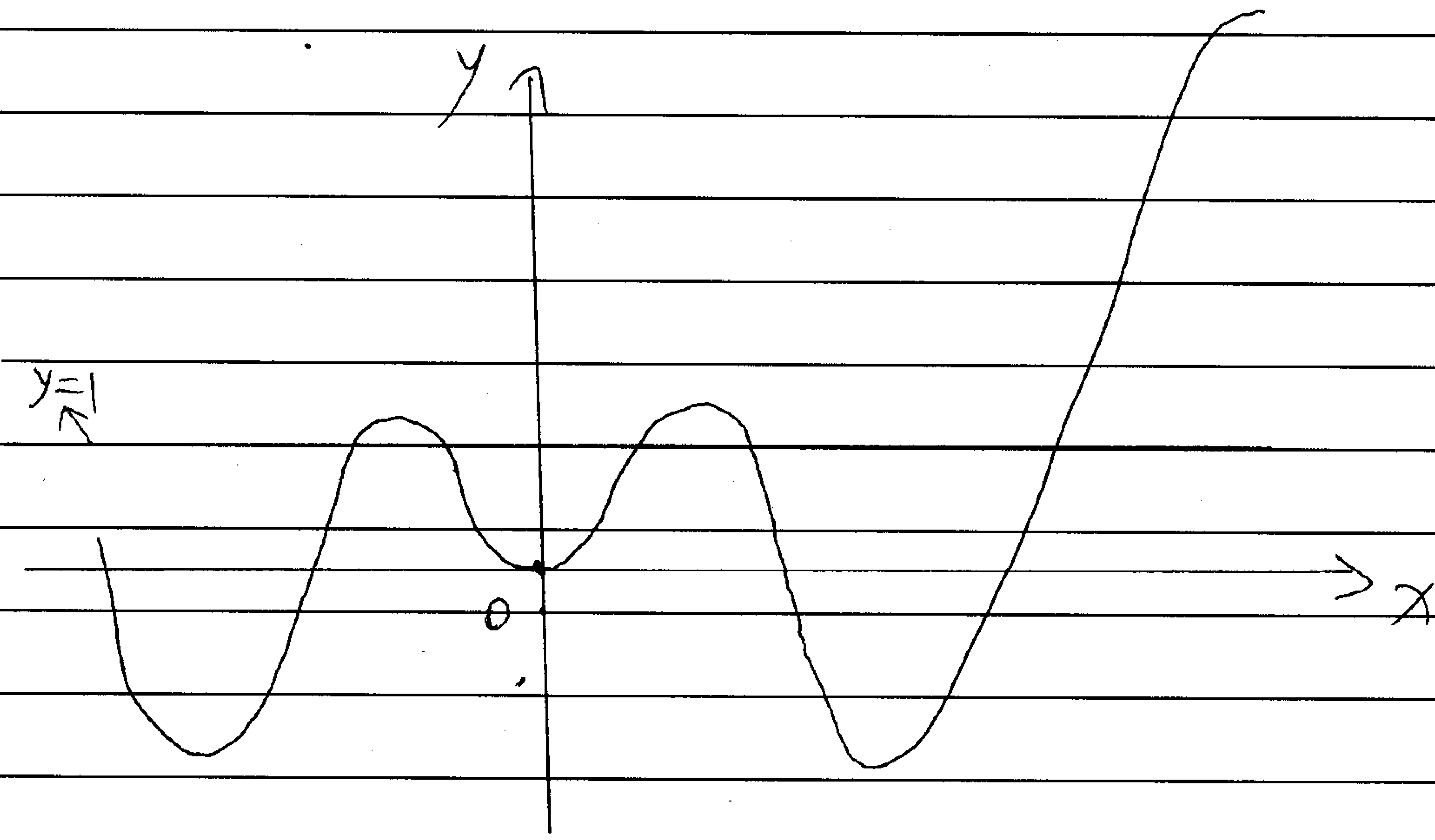
70

Let $g(x) = x \sin x - 1$ is continuous everywhere

Since $g(0) = 0 - 1 = -1 < 0$ & $g(2) = 2 \cdot \sin 2 - 1 > 2 \cdot \frac{1}{2} - 1 = 0$

By I.V.T., there's a $c \in [0, 2]$ s.t. $g(c) = 0$

There are infinitely many solutions of the equation $x \sin x = 1$




When $x \to \infty$ or $x \to -\infty$.

$y=1$ & $y = x \sin x$ would cross with each other infinitely many times.