HW01

2.2 − 72(b)

$$\lim_{x\to\infty}\left(\sqrt{x+54}-\sqrt{x}\right)=\lim_{x\to\infty}\left(\sqrt{x+54}-\sqrt{x}\right)\cdot\frac{\sqrt{x+54}+\sqrt{x}}{\sqrt{x+54}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{54}{\sqrt{x+54}+\sqrt{x}}=0$$

2.3 − 40

Since $\frac{x-1}{x}<\frac{\lfloor x\rfloor}{x}\leq 1$, $x\neq 0$

and $\lim_{x\to\infty}\frac{x-1}{x}=1 \Rightarrow \lim_{x\to\infty}\frac{\lfloor x\rfloor}{x}=1$ (By Sandwich Theorem)

$\lim_{x\to-\infty}\frac{x-1}{x}=1 \Rightarrow \lim_{x\to-\infty}\frac{\lfloor x\rfloor}{x}=1$

46

(a) We can't conclude anything about the value of f, g and h at $x=2$

Because limitation only describe the behavior around $x=2$, not exactly the behavior at $x=2$.

(b)

$f(2)=0$ is possible.

Ex: $f(x)=\begin{cases}2(x-2)-5, & x\neq 2\\ 0, & x=2\end{cases}$, $g(x)=\begin{cases}3(x-2)-5, & x<2\\ -1, & x=2\\ (x-2)-5, & x>2\end{cases}$

$h(x)=\begin{cases}(x-2)-5, & x<2\\ 1, & x=2\\ 3(x-2)-5, & x>2\end{cases}$

(c) $\lim_{x\to 2}f(x)$ can't be 0, or it will make a contradiction to the Sandwich theorem.

3. (a) "$x \to c^+$"

$\Rightarrow$ for all x in the interval $(c, c+\delta)$ for some $\delta > 0$

(b) "$x \to \infty$"

$\Rightarrow$ for all x in the interval $[M, \infty)$ for some $M \in \mathbb{R}$