Quiz 06

1. $\int_0^{\pi} \cos^2 x \sin^2 x \, dx = \int_0^{\pi} (\sin x \cos x)^2 \, dx = \int_0^{\pi} \left(\frac{\sin 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int_0^{\pi} \sin^2 2x \, dx = \frac{1}{4}\int_0^{\pi} \frac{1-\cos 4x}{2} \, dx$

$= \frac{1}{4}\left[\frac{x}{2} - \frac{1}{8}\sin 4x\right]_0^{\pi} = \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}$

2. $\int x^2 \sin x \, dx \qquad \left(\text{Let } \begin{cases} u = x^2 \\ dv = \sin x \, dx \end{cases}\right)$

$= -x^2\cos x - \int -2x\cos x \, dx$

$= -x^2\cos x + \int 2x\cos x \, dx \qquad \left(\text{Let } \begin{cases} u_1 = 2x \\ dv_1 = \cos x \, dx \end{cases}\right)$

$= -x^2\cos x + \left(2x\sin x - \int 2\sin x \, dx\right)$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

3. $\int \frac{x^2}{(x-1)(x+1)^2} dx = \int \frac{\frac{1}{4}}{x-1} dx + \int \frac{\frac{3}{4}}{x+1} dx + \int \frac{-\frac{1}{2}}{(x+1)^2} dx$

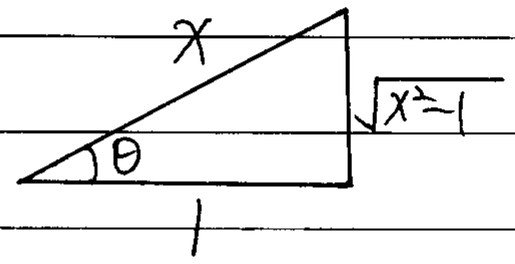
$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C$

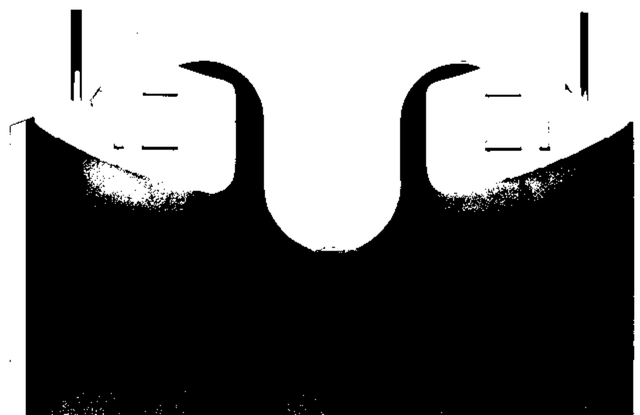
4. $\int \frac{dx}{x^2\sqrt{x^2-1}}, \quad x > 1 \qquad (\text{Let } x = \sec\theta)$

$= \int \frac{\sec\theta\cdot\tan\theta \, d\theta}{\sec^2\theta\sqrt{\sec^2\theta - 1}} = \int \frac{1}{\sec\theta} d\theta = \int \cos\theta \, d\theta$

$= \sin\theta + C$

$= \frac{\sqrt{x^2-1}}{x} + C$





5. $$\int_0^{\frac{\pi}{3}} \frac{1}{1-\sin x}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \frac{1+\sin x}{1-\sin^2 x}\,dx = \int_0^{\frac{\pi}{3}} \frac{1+\sin x}{\cos^2 x}\,dx$$

$$= \int_0^{\frac{\pi}{3}} \sec^2 x + \sec x\cdot\tan x\,dx$$

$$= \left[\tan x + \sec x\right]_0^{\frac{\pi}{3}} = \left[\sqrt{3}+2\right] - \left[0+1\right] = \sqrt{3}+1$$