QUIZ 5

1. $\lim\limits_{x\to\infty}\frac{\ln x}{x^{0.1}} = \lim\limits_{x\to\infty}\frac{\frac{1}{x}}{0.1x^{-0.9}} = \lim\limits_{x\to\infty}\frac{10}{x^{0.1}} = 0 \Rightarrow \ln x = o(x^{0.1})$ as $x \to \infty$

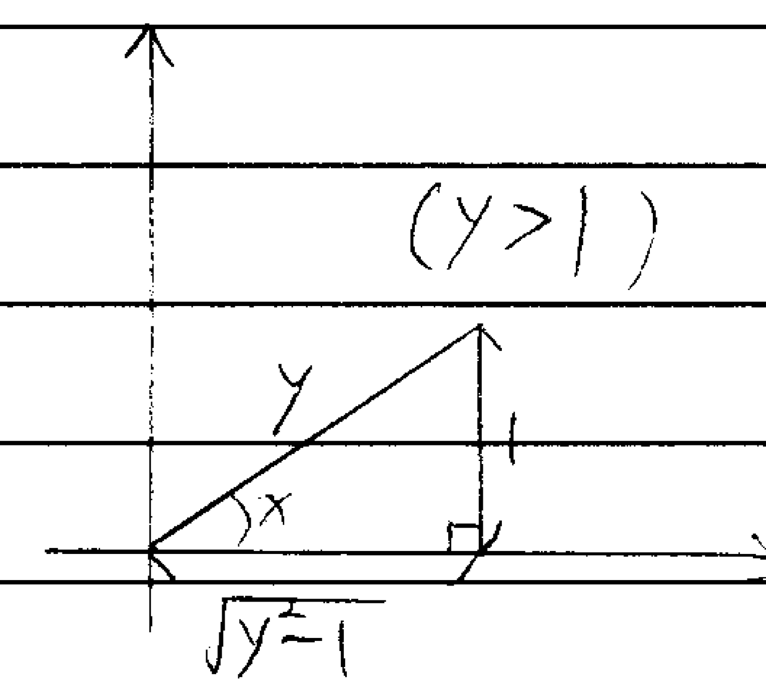
Hence it's true.

2. Show that $\frac{d\csc^{-1}y}{dy} = \frac{-1}{|y|\sqrt{y^2-1}}$, $|y|>1$

Pf: Let $\csc^{-1}y = x \Rightarrow \csc x = y \Rightarrow \frac{d\csc x}{dy} = 1$

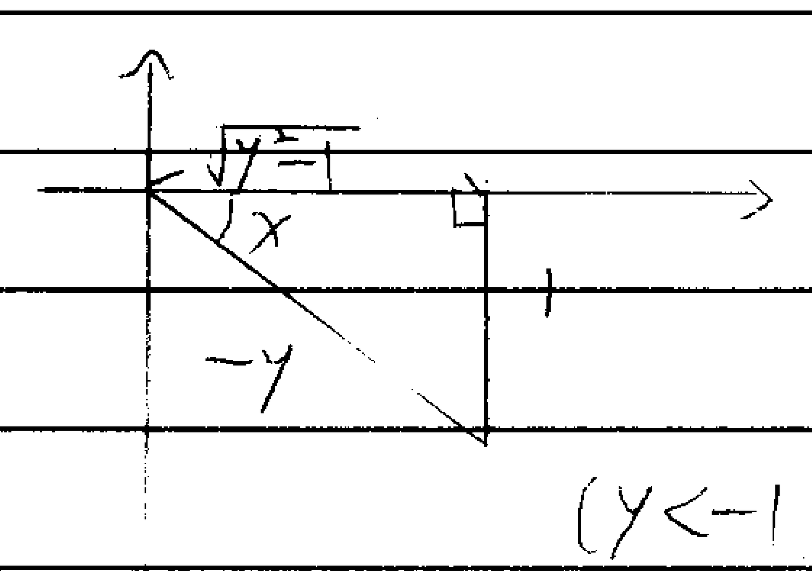
$\Rightarrow -\csc x\cdot\cot x\cdot\frac{dx}{dy} = 1$ ($\frac{dx}{dy}$ is what we want)

$\Rightarrow \frac{dx}{dy} = \frac{-1}{\csc x\cdot\cot x}$



When $y>1 \Rightarrow \csc x = y$ and $\cot x = \sqrt{y^2-1}$

When $y<-1 \Rightarrow \csc x = y$ and $\cot x = -\sqrt{y^2-1}$



Hence

$$\frac{d}{dy}(\csc^{-1}y) = \begin{cases} \frac{-1}{y\sqrt{y^2-1}}, & \text{if } y>1 \\ \frac{1}{y\sqrt{y^2-1}}, & \text{if } y<-1 \end{cases}$$

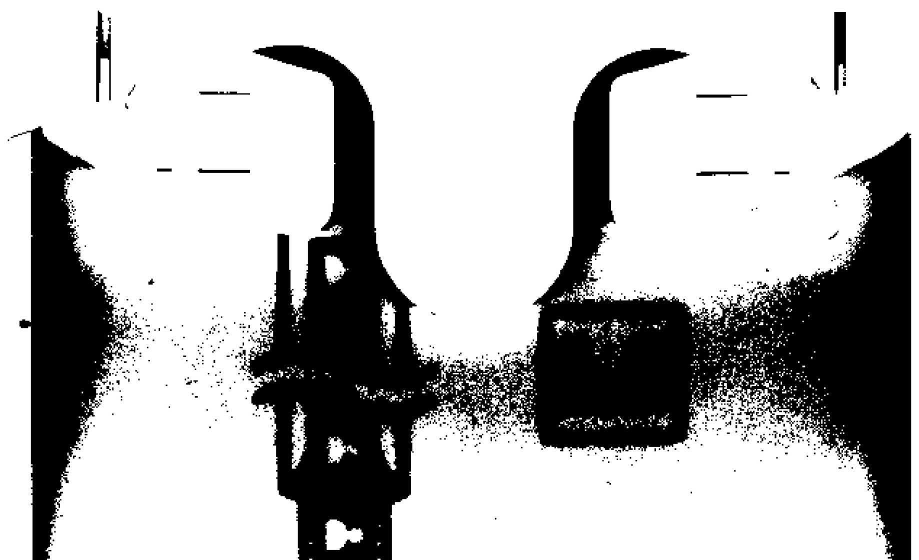
$\Rightarrow \frac{d}{dy}(\csc^{-1}y) = \frac{-1}{|y|\sqrt{y^2-1}}$, $|y|>1$

3. $\int_0^{0.6}\frac{s}{\sqrt{1-s^4}}\,ds = \int_0^{0.36}\frac{1}{2\sqrt{1-u^2}}\,du$, let $u=s^2 \Rightarrow du = 2s\,ds$

$= \frac{1}{2}\sin^{-1}u\Big|_0^{0.36} = \frac{1}{2}(\sin^{-1}0.36 - \sin^{-1}0) = \frac{1}{2}\sin^{-1}(0.36)$

4. $\int\tanh(2x)\,dx = \int\frac{\sinh(2x)}{\cosh(2x)}\,dx = \int\frac{1}{2u}\,du$, let $u=\cosh(2x) \Rightarrow du = 2\sinh(2x)\,dx$

$= \frac{1}{2}\ln u + C = \frac{1}{2}\ln\cosh(2x) + C$



5. Volume $= \int_0^1 \pi (f(y))^2 \, dy$

Area of surface $= \int_0^1 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2} \, dy = \int_0^1 2\pi f(y) \sqrt{1+\left(\frac{df}{dy}\right)^2} \, dy$