Quiz 4

1. (i) Let $P = \{0, 0.15, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$

$\Rightarrow \|P\| =$ longest subinterval length $= 0.15$

(ii) $$\int_0^1 f(x)\,dx = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(C_k)\cdot \Delta x_k$$

where $P = \{x_0, x_1, \cdots, x_n\}$ is a partition of $[0,1]$

$\Delta x_k = x_k - x_{k-1}$ and $C_k \in [x_{k-1}, x_k]$, $k = 1, 2, \cdots, n$

2. The Fundamental Theorem of Calculus, Part 1:

If f is continuous on $[a,b]$ then $F(x) = \int_a^x f(t)\,dt$ has a derivative at every point of $[a,b]$ and $\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

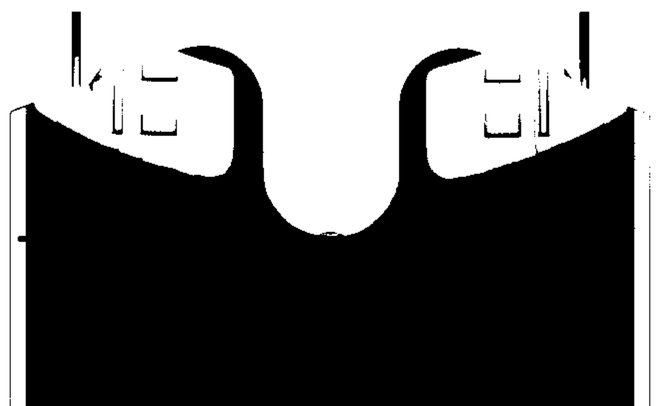
The Fundamental Theorem of Calculus, Part 2:

If f is continuous on $[a,b]$ and F is any antiderivative of f on $[a,b]$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

3. Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$$\Rightarrow \frac{d}{dx}\int_1^{x^2} \sin(t^3)\,dt = \frac{d}{du}\int_1^{u} \sin(t^3)\,dt \cdot \frac{du}{dx}$$

$$= \sin(u^3)\cdot 2x = 2x\sin(x^6)$$



4. Let $u = 4+3\sin\theta \Rightarrow du = 3\cos\theta\, d\theta$

$$\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{4+3\sin\theta}}\, d\theta = \int_4^{4+3\sin\frac{\pi}{2}} \frac{1}{3\sqrt{u}}\, du = \frac{2}{3}\sqrt{u}\Big|_4^{4+3\sin\frac{\pi}{2}} = \frac{2}{3}\left(\sqrt{4+3\sin\frac{\pi}{2}} - 2\right)$$

5. When $2 = y = f(x) = x^3 + 1 \Rightarrow x = 1$

$$\Rightarrow \left.\frac{df^{-1}(y)}{dy}\right|_{y=2} = \frac{1}{\left.\frac{df(x)}{dx}\right|_{x=1}} = \frac{1}{3\cdot 1^2} = \frac{1}{3}$$