Quiz 03

1. $y = f(x) = x + \frac{1}{x}$

$\because \lim_{x \to 0^+} f(x) = \infty \Rightarrow x=0$ is a vertical asymptote of $f(x)$

$\because f(x) \approx x$ for large $x \Rightarrow y = x$ is an oblique asymptote of $f(x)$

2. $f(x) = x^3 - \frac{3}{x}$

$f(x) \approx x^3$ when $|x|$ is large $\Rightarrow x^3$ dominates when $|x|$ is large

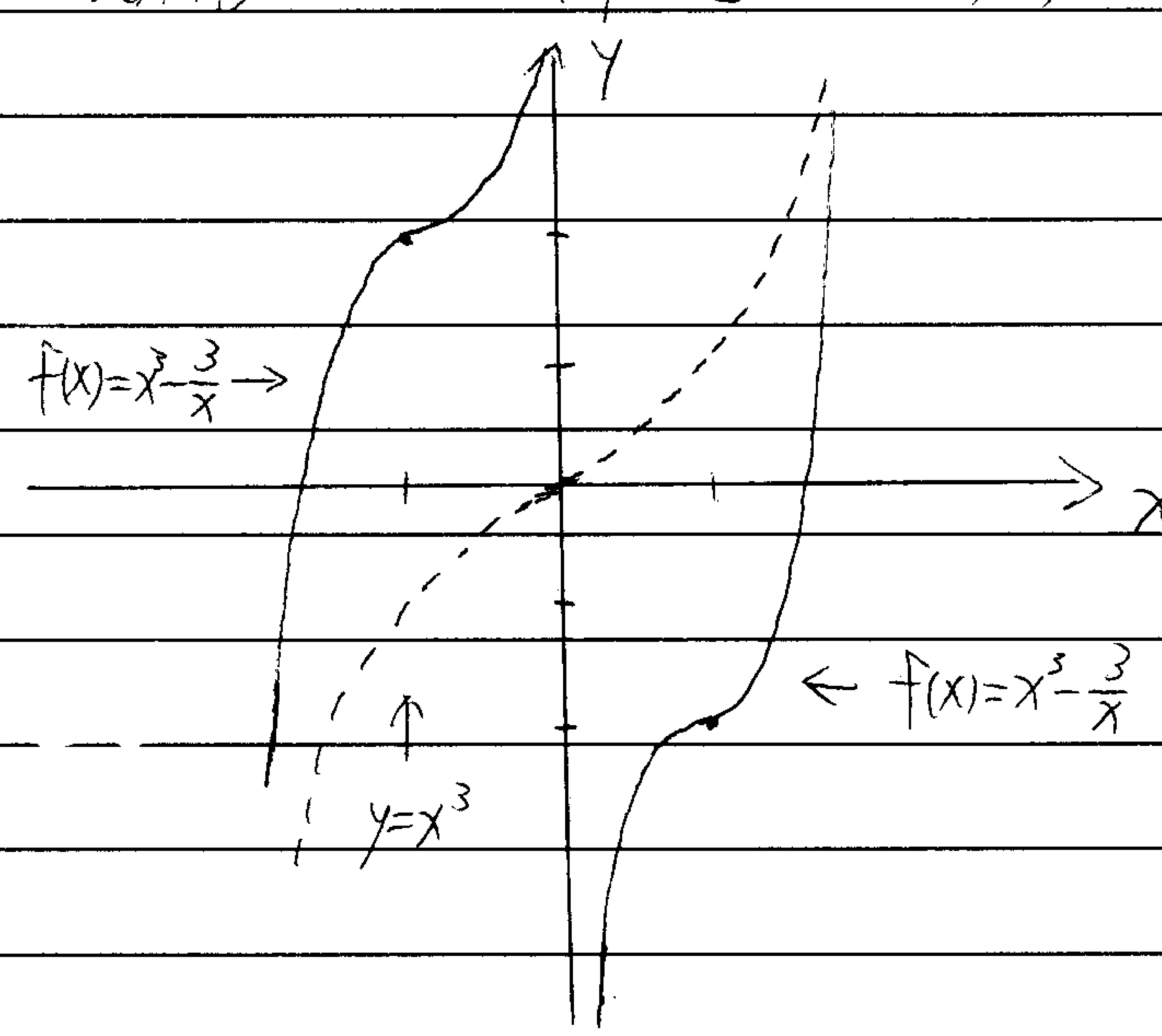
$f(x) \approx -\frac{3}{x}$ when x is close to $0 \Rightarrow -\frac{3}{x}$ dominates when x is close to 0

$\because \lim_{x \to 0^+} f(x) = -\infty \Rightarrow x=0$ is a vertical asymptote of $f(x)$

$f'(x) = 3x^2 + \frac{3}{x^2} \Rightarrow$ critical point : $x = 0$

$f''(x) = 6x - \frac{6}{x^3} = \frac{6}{x^3}(x^4-1) = \frac{6}{x^3}(x-1)(x+1)(x^2+1) \Rightarrow$ inflection points : $x = -1, 0, 1$

f'	+	+	+	+
f''	−	+	−	+
	$x<-1$	$-1<x<0$	$0<x<1$	$x>1$



$f(-1) = 2$ and $f(1) = -2$

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3. Let $P=(a,b)$ be a point on the curve $\{(x,y)\,|\,y=x^2\} \Rightarrow b=a^2$

and D be the distance from $(0,\frac{3}{2})$ to P

$\Rightarrow D^2=(a-0)^2+(b-\frac{3}{2})^2=a^2+(b-\frac{3}{2})^2=a^2+(a^2-\frac{3}{2})^2$

$\Rightarrow \frac{dD}{da}=2a+2(a^2-\frac{3}{2})\cdot 2a=2a(1+2a^2-3)=4a(a^2-1)=4a(a-1)(a+1)$

$\Rightarrow$ critical points : $a=0,1,-1 \Rightarrow$

$\frac{dD}{da}$	$-$	$+$	$-$	$+$
	$a<-1$	$-1<a<0$	$0<a<1$	$a>1$

$\Rightarrow D$ has local minimum at $a=-1$ and $a=1$

$D(-1)=\frac{5}{4}$, $D(1)=\frac{5}{4} \Rightarrow$ minimal distance $=\frac{\sqrt{5}}{2}$

4. (i) Let $f_1(x)=x^4 \Rightarrow f_1'(0)=0$ and $f_1''(0)=0$

Moreover, $f_1'(x)>0$ when $x>0$, $f_1'(x)<0$ when $x<0$ $\Rightarrow f_1$ has a local minimum at $x=0$

(ii) Let $f_2(x)=-x^4 \Rightarrow f_2'(0)=0$ and $f_2''(0)=0$

Moreover, $f_2'(x)<0$ when $x>0$, $f_2'(x)>0$ when $x<0$ $\Rightarrow f_2$ has a local maximum at $x=0$

(iii) Let $f_3(x)=x^3 \Rightarrow f_3'(0)=0$ and $f_3''(0)=0$

Moreover, $f_3'(x)>0$ when $x\neq 0$ $\Rightarrow f_3(0)$ is not a local minimum or a local maximum.

5. (i) Intermediate Value Theorem:

If f is continuous on $[a,b]$, then $\forall M$ between $f(a)$ & $f(b)$, $\exists\ c \in [a,b]$ s.t. $f(c)=M$.

(ii) Mean Value Theorem:

If f is continuous on $[a,b]$, and differentiable on (a,b), then $\exists\ c \in (a,b)$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$

(iii) Rolle's Theorem:

If f is continuous on $[a,b]$, and differentiable on (a,b), and $f(a)=f(b)=0$, then $\exists\ c \in (a,b)$ s.t. $f'(c)=0$

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