Quiz 2

1. $y=\sqrt{x+\sqrt{x+\sqrt{x}}}=(x+(x+x^{\frac{1}{2}})^{\frac{1}{2}})^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx}=\left[\frac{1}{2}(x+(x+x^{\frac{1}{2}})^{\frac{1}{2}})^{-\frac{1}{2}}\right]\cdot\left[1+\frac{1}{2}(x+x^{\frac{1}{2}})^{-\frac{1}{2}}\cdot(1+\frac{1}{2}x^{-\frac{1}{2}})\right]$

2. $x^4+y^4=2 \Rightarrow 4x^3+4y^3\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x^3}{y^3}$

$\Rightarrow 12x^2+12y^2\left(\frac{dy}{dx}\right)^2+4y^3\frac{d^2y}{dx^2}=0$

Hence $\frac{dy}{dx}(1,1)=-1$ and $12+12\cdot1\cdot(-1)^2+4\cdot1\cdot\frac{d^2y}{dx^2}=0 \Rightarrow \frac{d^2y}{dx^2}=-6$

3. $f(x)=\frac{1}{1+\tan x} \Rightarrow f'(x)=-\frac{\sec^2 x}{(1+\tan x)^2}$

$\Rightarrow L(x)=f(0)+f'(0)(x-0)=1+(-1)x=1-x$

4. Let $f(x)=(1+x)^{\frac{1}{2}} \Rightarrow (1.001)^{\frac{1}{2}}=f(0.001)$

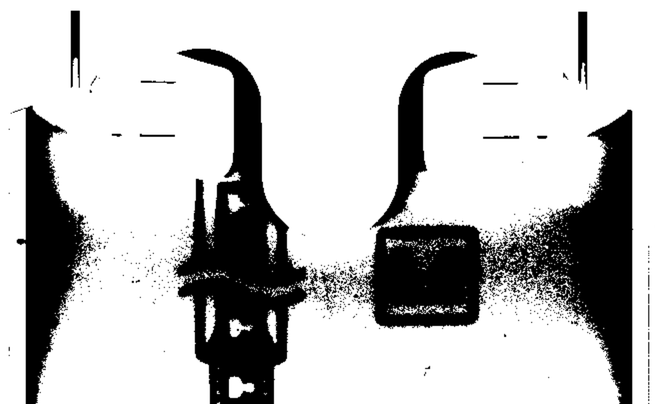
Linear approximation of $f(x)$ near $x=0$: $L(x)=1+\frac{1}{2}x$

Approximate value of $\sqrt{1.001} \approx L(0.001)=1.0005$

Since $f''(x)=-\frac{1}{4}(1+x)^{-\frac{3}{2}}$

$|f(0.001)-L(0.001)| \le \frac{1}{2}\max_{\xi\in[0,0.001]}|f''(\xi)|(0.001-0)^2=\frac{1}{2}\cdot\frac{1}{4}\cdot10^{-6}=\frac{1}{8}\cdot10^{-6}$

Hence the error is smaller than 10^{-6}



5. $f(x+\Delta x) - f(x) = f'(x_0)\Delta x + \alpha\Delta x$

$\Rightarrow \alpha = \frac{f(x+\Delta x)-f(x)}{\Delta x} - f'(x_0)$ when $\Delta x \neq 0$

If f is differentiable at $x \Rightarrow \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = f'(x)$

$\Rightarrow \lim_{\Delta x \to 0} \alpha = f'(x) - f'(x_0)$