Quiz 1

1. Given $\epsilon > 0$, $\exists\ \delta > 0$ such that for all x

$$0 < |x-1| < \delta \Rightarrow \left|\frac{1}{x} - 1\right| < \epsilon$$

2.

$$\left|\frac{1}{x} - 1\right| < 0.1 \iff -0.1 < \frac{1}{x} - 1 < 0.1$$

$$\iff 0.9 < \frac{1}{x} < 1.1$$

$$\iff \frac{1}{1.1} < x < \frac{1}{0.9} \iff \underbrace{-\frac{0.1}{1.1}}_{-\frac{1}{11}} < x-1 < \underbrace{\frac{0.1}{0.9}}_{\frac{1}{9}}$$

Hence we take $\delta = \frac{1}{11}$

3.

(a) If $f(x)$ is a continuous function on $[a, b]$,

then $\forall\ M \in [f(a), f(b)]$, $\exists\ C \in [a, b]$ such that $f(C) = M$

(b)

Let $f(x) = x - 1 - \cos x$

Since $f(1) = 1 - 1 - \cos 1 = -\cos 1 < 0$

& $f(2) = 2 - 1 - \cos 2 = 1 - \cos 2 > 0$

By the Intermediate Value Theorem, we can find $t \in [1, 2]$

such that $f(t) = 0$

4.

(a) $\lim_{x \to 0} f(x) = 1$

$\Rightarrow$ Given $\epsilon > 0$, $\exists\, \delta > 0$ such that for all x

$-\delta < x - 0 < 0 \quad \Rightarrow |f(x) - 1| < \epsilon$

(b) $\lim_{x \to \infty} f(x) = 1$

$\Rightarrow$ Given $\epsilon > 0$, $\exists\, M \in \mathbb{R}$ such that for all x

$x > M \Rightarrow |f(x) - 1| < \epsilon$

5.

$f(x) = \sqrt{x}$

$$\Rightarrow f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$$

$$= \lim_{x \to 2} \frac{(\sqrt{x} - \sqrt{2})(\sqrt{x} + \sqrt{2})}{(x-2)(\sqrt{x} + \sqrt{2})} = \lim_{x \to 2} \frac{x - 2}{(x-2)(\sqrt{x} + \sqrt{2})} = \lim_{x \to 2} \frac{1}{\sqrt{x} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$