(a) $\int \frac{1}{e^{2x}-e^{-x}}\,dx = \int \frac{e^x}{e^{3x}-1}\,dx$ (Let $y=e^x$)

$$= \int \frac{1}{y^3-1}\,dy = \int \frac{1}{(y-1)(y^2+y+1)}\,dy = \int \frac{\frac{1}{3}}{y-1}\,dy + \int \frac{-\frac{1}{3}y-\frac{2}{3}}{y^2+y+1}\,dy$$

$$= \frac{1}{3}\ln|y-1| - \frac{1}{3}\int \frac{y+2}{(y+\frac{1}{2})^2+\frac{3}{4}}\,dy$$ (Let $u=y+\frac{1}{2}$)

$$= \frac{1}{3}\ln|e^x-1| - \frac{1}{3}\int \frac{u+\frac{3}{2}}{u^2+\frac{3}{4}}\,du$$

$$\int \frac{u+\frac{3}{2}}{u^2+\frac{3}{4}}\,du = \int \frac{u}{u^2+\frac{3}{4}}\,du + \int \frac{\frac{3}{2}}{u^2+\frac{3}{4}}\,du = \frac{1}{2}\ln\left|u^2+\frac{3}{4}\right| + \frac{3}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}u\right)$$

$$\Rightarrow \int \frac{1}{e^{2x}-e^{-x}}\,dx = \frac{1}{3}\ln|e^x-1| - \frac{1}{6}\ln\left|\left(e^x+\frac{1}{2}\right)^2+\frac{3}{4}\right| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2}{\sqrt{3}}\left(e^x+\frac{1}{2}\right)\right) + C$$

(b)

$$\int \frac{1}{\sin^2 x\cos^2 x}\,dx = \int \frac{4}{(2\sin x\cos x)^2}\,dx = 4\int \frac{1}{(\sin 2x)^2}\,dx = 4\int \csc^2 2x\,dx$$

$$= -2\cot(2x) + C$$

(c)

$$\int x^x(1+\ln x)\,dx \qquad (\text{Let } u = x^x,\ du = x^x(1+\ln x)\,dx)$$

$$= \int 1\,du = u + C = x^x + C$$

(d)

$$\int \frac{x\sin x}{\cos^2 x}\,dx = \int x\cdot\frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}\,dx = \int x\sec x\tan x\,dx$$

$$= x\sec x - \int \sec x\,dx \qquad \left(\begin{array}{l} u = x \\ dv = \sec x\tan x\,dx \end{array}\right)$$

$$= x\sec x - \ln|\sec x + \tan x| + C$$

(e)

$$\int_0^{\pi}\sqrt{1+\sin x}\,dx = \int_0^{\pi}\sqrt{\frac{(1+\sin x)(1-\sin x)}{1-\sin x}}\,dx = \int_0^{\pi}\sqrt{\frac{1-\sin^2 x}{1-\sin x}}\,dx$$

$$= \int_0^{\pi}\frac{|\cos x|}{\sqrt{1-\sin x}}\,dx = \int_0^{\frac{\pi}{2}}\frac{\cos x}{\sqrt{1-\sin x}}\,dx + \int_{\frac{\pi}{2}}^{\pi}\frac{-\cos x}{\sqrt{1-\sin x}}\,dx \qquad (\text{Let } u = \sin x)$$

$$= \int_0^1\frac{1}{\sqrt{1-u}}\,du - \int_1^0\frac{1}{\sqrt{1-u}}\,du = 2\int_0^1\frac{1}{\sqrt{1-u}}\,du$$

$$= 2\lim_{b\to 1^-}\left[-2\sqrt{1-u}\right]_0^b = 2\lim_{b\to 1^-}\left[-2\sqrt{1-b}+2\right] = 2\cdot 2 = 4$$

8.6-38

$$\int_0^1 \frac{1}{t-\sin t}\,dt$$

Since $t \geq \sin t \geq 0$ for $0 \leq t \leq 1 \Rightarrow 0 \leq t-\sin t \leq t$

$$\Rightarrow 0 \leq \frac{1}{t} \leq \frac{1}{t-\sin t}$$

And $\int_0^1 \frac{1}{t}\,dt = \lim_{b\to 0^+}\left[\ln|t|\right]_b^1 = \lim_{b\to 0^+}\left[-\ln b\right] = \infty$ diverges

$\Rightarrow \int_0^1 \frac{1}{t-\sin t}\,dt$ diverges

54

$$\int_2^\infty \frac{1}{\ln x}\,dx$$

Since $0 < \frac{1}{x} < \frac{1}{\ln x}$ for $x \geq 2$, and $\int_2^\infty \frac{1}{x}\,dx$ diverges

$\Rightarrow \int_2^\infty \frac{1}{\ln x}\,dx$ diverges

56 $\int_{e^e}^{\infty} \ln(\ln x)\,dx$

Since $\ln(\ln x) \geq 1$ for $x \geq e^e$, and $\int_{e^e}^{\infty} 1\,dx$ diverges

$\Rightarrow \int_{e^e}^{\infty} \ln(\ln x)\,dx$ diverges

58 $\int_{1}^{\infty} \frac{1}{e^x - 2^x}\,dx$

Since $\lim_{x\to\infty} \frac{\frac{1}{e^x-2^x}}{\frac{1}{e^x}} = \lim_{x\to\infty} \frac{e^x}{e^x-2^x} = \lim_{x\to\infty} \frac{1}{1-(\frac{2}{e})^x} = 1$

and $\int_{1}^{\infty} \frac{1}{e^x}\,dx = \lim_{b\to\infty} \left[-\frac{1}{e^x}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{e^b}+\frac{1}{e}\right) = \frac{1}{e}$ converges

$\Rightarrow \int_{1}^{\infty} \frac{1}{e^x-2^x}\,dx$ converges

60 $\int_{-\infty}^{\infty} \frac{1}{e^x+e^{-x}}\,dx = 2\int_{0}^{\infty} \frac{1}{e^x+e^{-x}}\,dx$

Since $0 \leq \frac{1}{e^x+e^{-x}} < \frac{1}{e^x}$ for $x>0$, and $\int_{0}^{\infty} \frac{1}{e^x}\,dx$ converges

$\Rightarrow \int_{0}^{\infty} \frac{1}{e^x+e^{-x}}\,dx$ converges $\Rightarrow \int_{-\infty}^{\infty} \frac{1}{e^x+e^{-x}}\,dx$ converges

71. $\int_{0}^{\infty} \frac{2x}{x^2+1}\,dx = \int_{1}^{\infty} \frac{1}{y}\,dy \qquad (\text{Let } y = x^2+1)$

$= \lim_{b\to\infty} \ln|y|\Big|_1^b = \lim_{b\to\infty} \ln b = \infty$ diverges

But $\lim_{b\to\infty} \int_{-b}^{b} \frac{2x}{x^2+1}\,dx = \lim_{b\to\infty} \int_{b^2+1}^{b^2+1} \frac{1}{y}\,dy = 0$

Hence $\int_{-\infty}^{\infty} f(x)\,dx$ may not equal $\lim_{b\to\infty}\int_{-b}^{b} f(x)\,dx$

73. $\int_{-1}^{3} \frac{1}{x^2}\,dx = \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{3} \frac{1}{x^2}\,dx$

And $\int_{-1}^{0} \frac{1}{x^2}\,dx = \lim_{b \to 0^-} \left[-\frac{1}{x}\right]_{-1}^{b} = \lim_{b \to 0^-} \left[-\frac{1}{b} - 1\right] = \infty$ diverges

$\Rightarrow \int_{-1}^{3} \frac{1}{x^2}\,dx$ diverges

The calculation in text book is wrong!