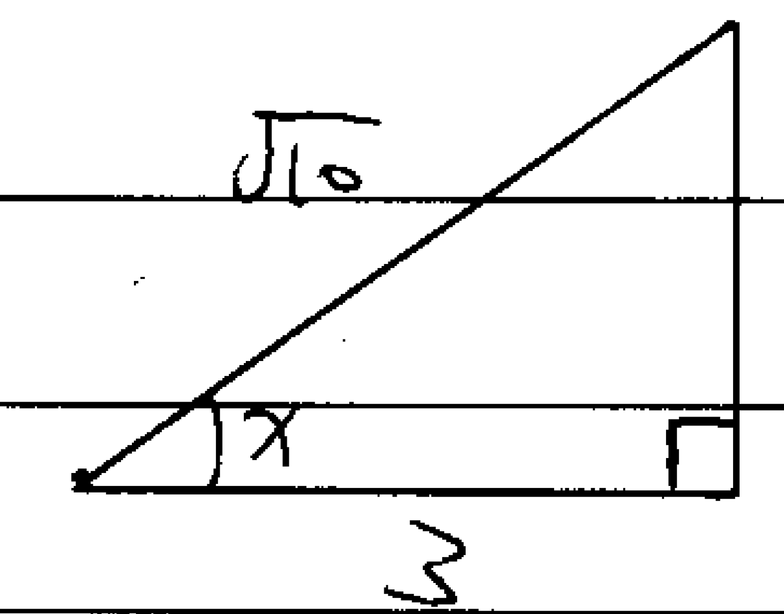
8.4 - 14

$$\int_0^{\ln 4} \frac{e^t}{\sqrt{e^{2t}+9}}\,dt \qquad \left(e^t = 3\tan x \Rightarrow e^t\,dt = 3\sec^2 x\,dx \right)$$

$$= \int_{t=0}^{t=\ln 4} \frac{3\sec^2 x}{\sqrt{9\tan^2 x+9}}\,dx = \int_{t=0}^{t=\ln 4} \frac{3\sec^2 x}{3\sec x}\,dx = \int_{t=0}^{t=\ln 4} \sec x\,dx$$

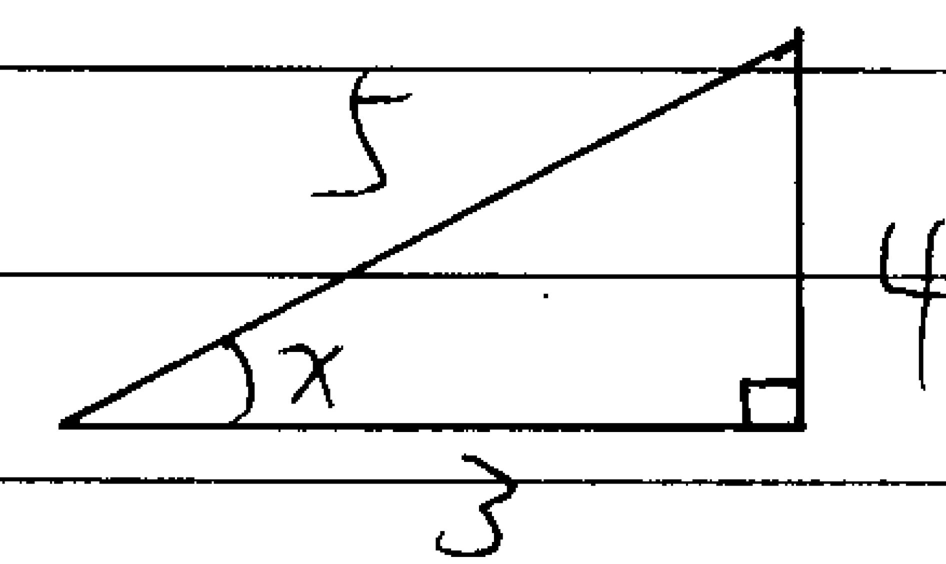
$$= \ln\left|\sec x + \tan x\right| \Big|_{t=0}^{t=\ln 4}$$

$$\left(t=0 \Rightarrow \tan x = \frac{1}{3} \Rightarrow \sec x = \frac{\sqrt{10}}{3} \right)$$



$$= \ln 3 - \ln\left(\frac{\sqrt{10}+1}{3}\right)$$

$$\left(t=\ln 4 \Rightarrow \tan x = \frac{4}{3} \Rightarrow \sec x = \frac{5}{3} \right)$$



$$= \ln\left(\frac{9}{\sqrt{10}+1}\right)$$

Ch8 – 130

$$\frac{x^3+2}{4-x^2} = -x + \frac{4x+2}{4-x^2} = -x + \frac{\frac{5}{2}}{2-x} - \frac{\frac{3}{2}}{2+x}$$

$$\Rightarrow \int \frac{x^3+2}{4-x^2}\,dx = -\frac{x^2}{2} - \frac{5}{2}\ln|x-2| - \frac{3}{2}\ln|x+2| + C$$

132

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\sin\sqrt{x} + C \qquad (\text{Let } u=\sqrt{x})$$

134

$$\int \frac{t-1}{\sqrt{t^2-2t}}\,dt = \sqrt{t^2-2t} + C \qquad (\text{Let } u = t^2-2t)$$

136

$$\int e^t \cos e^t\,dt = \sin(e^t) + C \qquad (\text{Let } u = e^t)$$

138

$$\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta = \tan\theta - \theta + C \qquad (\text{Use } \sin^2\theta = 1-\cos^2\theta)$$

140

$$\int \frac{\cos x}{1+\sin^2 x}\,dx = \tan^{-1}(\sin x) + C \qquad (\text{Let } u = \sin x)$$

142

$$\int_2^\infty \frac{1}{(x-1)^2}\,dx = \lim_{b\to\infty} \int_2^b \frac{1}{(x-1)^2}\,dx = \lim_{b\to\infty}\left[\frac{-1}{x-1}\right]_2^b = 1$$

144

$$\int \frac{d\theta}{\sqrt{1+\sqrt{\theta}}} = 4\left[\frac{(\sqrt{1+\sqrt{\theta}})^3}{3} - \sqrt{1+\sqrt{\theta}}\right] + C \qquad (\text{Let } \sqrt{\theta} = \tan^2 x)$$

146

$$\int \frac{x^5}{x^4-16}\,dx = \int x\,dx + \int \frac{16x}{x^4-16}\,dx = \frac{x^2}{2} + 8\int \frac{1}{y^2-4^2}\,dy \qquad (y = x^2)$$

$$= \frac{x^2}{2} + \left(\int \frac{1}{y-4}\,dy - \int \frac{1}{y+4}\,dy\right) = \frac{x^2}{2} + \ln|x^2-4| - \ln|x^2+4| + C$$

148 $\displaystyle\int \frac{1}{\theta^2-2\theta+4}\,d\theta = \int \frac{1}{(\theta-1)^2+3}\,d\theta = \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\theta-1}{\sqrt{3}}\right)+C$

150 $\displaystyle\int \frac{1}{(r+1)\sqrt{r^2+2r}}\,dr = \int \frac{1}{u\sqrt{u^2-1}}\,du$ (Let $u=r+1$)

$$= \sec^{-1}|u|+C = \sec^{-1}|r+1|+C$$

152 $\displaystyle\int \frac{y}{4+y^4}\,dy = \int \frac{\frac{1}{2}}{4+u^2}\,du$ (Let $u=y^2$)

$$= \frac{1}{4}\tan^{-1}\left(\frac{u}{2}\right)+C = \frac{1}{4}\tan^{-1}\left(\frac{y^2}{2}\right)+C$$

154 $\displaystyle\int \frac{1}{(x^2-1)^2}\,dx = \int \frac{1}{(x-1)^2(x+1)^2}\,dx$

$$= \int \frac{-\frac{1}{4}}{x-1} + \frac{\frac{1}{4}}{(x-1)^2} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{4}}{(x+1)^2}\,dx$$

$$= \frac{1}{4}\left(-\ln|x-1| - \frac{1}{x-1} + \ln|x+1| - \frac{1}{x+1}\right)+C$$

156 $\displaystyle\int (15)^{2x+1}\,dx = \frac{1}{2\ln 15}(15)^{2x+1}+C$

158 $\displaystyle\int \frac{\sqrt{1-v^2}}{v^2}\,dv = \int \frac{\cos\theta\cdot\cos\theta\,d\theta}{\sin^2\theta}$ (Let $v=\sin\theta$)

$$= \int \frac{1-\sin^2\theta}{\sin^2\theta}\,d\theta = \int \csc^2\theta\,d\theta - \int 1\,d\theta$$

$$= -\cot\theta - \theta + C$$

$$= -\frac{\sqrt{1-v^2}}{v} - \sin^{-1}v + C$$

160 $\int \ln(\sqrt{x-1})\,dx = \int (\ln y)\cdot 2y\,dy \qquad (\text{Let } x = y^2+1)$

$= y^2 \ln y - \int y\,dy \qquad \left(\begin{array}{ll} u = \ln y, & dv = 2y\,dy \\ \Rightarrow du = \frac{1}{y}dy, & v = y^2 \end{array} \right)$

$= y^2 \ln y - \frac{1}{2}y^2 + C$

$= (x-1)\ln(\sqrt{x-1}) - \frac{1}{2}(x-1) + C$

162 $\int \frac{x}{\sqrt{8-2x^2-x^4}}\,dx = \int \frac{x}{\sqrt{9-(x^2+1)^2}}\,dx \qquad (\text{Let } y = x^2+1)$

$= \frac{1}{2}\int \frac{1}{\sqrt{9-y^2}}\,dy$

$= \frac{1}{2}\sin^{-1}\left(\frac{y}{3}\right) + C = \frac{1}{2}\sin^{-1}\left(\frac{x^2+1}{3}\right) + C$

164 $\int x^3 e^{x^2}\,dx = \frac{1}{2}\int t e^t\,dt \qquad (\text{Let } t = x^2)$

$= \frac{1}{2}\left(te^t - \int e^t\,dt\right) + C \qquad \left(\begin{array}{ll} u = t, & dv = e^t\,dt \\ \Rightarrow du = dt, & v = e^t \end{array} \right)$

$= \frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C$

166 $\int_0^{\frac{\pi}{10}} \sqrt{1+\cos 5\theta}\,d\theta = \int_0^{\frac{\pi}{10}} \sqrt{2}\cos\left(\frac{5}{2}\theta\right)\,d\theta \qquad \left(\cos 5\theta = 2\cos^2\left(\frac{5}{2}\theta\right) - 1\right)$

$= \frac{2\sqrt{2}}{5}\sin\left(\frac{5}{2}\theta\right)\Big|_0^{\frac{\pi}{10}}$

$= \frac{2}{5}$

168 $\displaystyle\int \frac{\tan^{-1}x}{x^2}\,dx = -\frac{\tan^{-1}x}{x} + \int \frac{1}{x(1+x^2)}\,dx \qquad \left(\begin{array}{ll} u = \tan^{-1}x \ ; & dv = \frac{1}{x^2}\,dx \\ \Rightarrow du = \frac{1}{1+x^2}\,dx \ , & v = -\frac{1}{x} \end{array} \right)$

$$= -\frac{\tan^{-1}x}{x} + \int \frac{1}{x}\,dx - \int \frac{x}{1+x^2}\,dx$$

$$= -\frac{\tan^{-1}x}{x} + \ln|x| - \frac{1}{2}\ln(1+x^2) + C$$

170 $\displaystyle\int \frac{e^t}{e^{2t}+3e^t+2}\,dt = \int \frac{1}{u^2+3u+2}\,du \qquad (\text{Let } u = e^t)$

$$= \int \frac{1}{(u+1)(u+2)}\,du = \int \frac{1}{u+1}\,du - \int \frac{1}{u+2}\,du$$

$$= \ln|u+1| - \ln|u+2| + C$$

$$= \ln\left|\frac{e^t+1}{e^t+2}\right| + C$$

172 $\displaystyle\int \frac{1-\cos 2x}{1+\cos 2x}\,dx = \int \frac{2\sin^2 x}{2\cos^2 x}\,dx = \int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$

$$= \tan x - x + C$$

174 $\displaystyle\int \frac{\cos x}{\sin^3 x - \sin x}\,dx = \int \frac{\cos x}{\sin x(\sin^2 x - 1)}\,dx = \int \frac{\cos x}{\sin x(-\cos^2 x)}\,dx = -\int \frac{1}{\sin x \cos x}\,dx$

$$= -\int \frac{2}{\sin 2x}\,dx \qquad (\sin 2x = 2\sin x \cos x)$$

$$= -2\int \csc 2x\,dx = \ln|\csc 2x + \cot 2x| + C$$

176 $\displaystyle\int \frac{x^2-x+2}{(x^2+2)^2}\,dx = \int \frac{1}{x^2+2}\,dx - \int \frac{x}{(x^2+2)^2}\,dx$

$\displaystyle = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{2}(x^2+2)^{-1} + C$

178 $\displaystyle\int \tan^3 t\,dt = \int \tan t\,(\sec^2 t - 1)\,dt \qquad (\tan^2 t = \sec^2 t - 1)$

$\displaystyle = \int \tan t \cdot \sec^2 t\,dt - \int \tan t\,dt$

$\displaystyle = \frac{\tan^2 t}{2} + \ln|\cos t| + C$

180 $\displaystyle\int \frac{3+\sec^2 x+\sin x}{\tan x}\,dx = \int 3\cot x\,dx + \int \frac{\sec^2 x}{\tan x}\,dx + \int \cos x\,dx$

$\displaystyle = 3\ln|\sin x| + \ln|\tan x| + \sin x + C$

182 $\displaystyle\int \frac{1}{(2x-1)\sqrt{x^2-x}}\,dx = \int \frac{1}{(2x-1)\sqrt{(x-\frac{1}{2})^2-\frac{1}{4}}}\,dx = \int \frac{2}{(2x-1)\sqrt{(2x-1)^2-1}}\,dx \qquad (\text{Let } u = 2x-1)$

$\displaystyle = \int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1}|u| + C$

$\displaystyle = \sec^{-1}|2x-1| + C$

184 $\displaystyle\int e^{\theta}\sqrt{3+4e^{\theta}}\,d\theta = \frac{1}{4}\int \sqrt{u}\,du \qquad (\text{Let } u = 3+4e^{\theta})$

$\displaystyle = \frac{1}{6}u^{\frac{3}{2}} + C$

$\displaystyle = \frac{1}{6}(3+4e^{\theta})^{\frac{3}{2}} + C$

186 $$\int \frac{1}{\sqrt{e^{2v}-1}}\,dv = \int \frac{\frac{1}{u}}{\sqrt{u^2-1}}\,du \qquad (\text{Let } u = e^v)$$

$$= \int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1}|u| + C = \sec^{-1}(e^v) + C$$

188 $$\int x^5 \sin x\,dx$$

differentiate		integrate
x^5	$(+)$	$\sin x$
$5x^4$	$(-)$	$-\cos x$
$20x^3$	$(+)$	$-\sin x$
$60x^2$	$(-)$	$\cos x$
$120x$	$(+)$	$\sin x$
120	$(-)$	$-\cos x$
0		$-\sin x$

$$= -x^5\cos x + 5x^4\sin x + 20x^3\cos x$$
$$-60x^2\sin x - 120\cos x + 120\sin x + C$$

190 $$\int \frac{4x^3-20x}{x^4-10x^2+9}\,dx = \int \frac{1}{u}\,du \qquad (\text{Let } u = x^4-10x^2+9)$$

$$= \ln|u| + C = \ln|x^4-10x^2+9| + C$$

192 $$\int \frac{t+1}{(t^2+2t)^{\frac{2}{3}}}\,dt = \frac{1}{2}\int \frac{1}{u^{\frac{2}{3}}}\,du \qquad (\text{Let } u = t^2+2t)$$

$$= \frac{3}{2}u^{\frac{1}{3}} + C = \frac{3}{2}(t^2+2t)^{\frac{1}{3}} + C$$

194 $$\int \frac{1}{t(1+\ln t)\sqrt{(\ln t)(2+\ln t)}}\,dt = \int \frac{1}{u\sqrt{(u-1)(u+1)}}\,du \qquad (\text{Let } u = 1+\ln t)$$

$$= \int \frac{1}{u\sqrt{u^2-1}}\,du = \sec^{-1}|u| + C$$

$$= \sec^{-1}|1+\ln t| + C$$