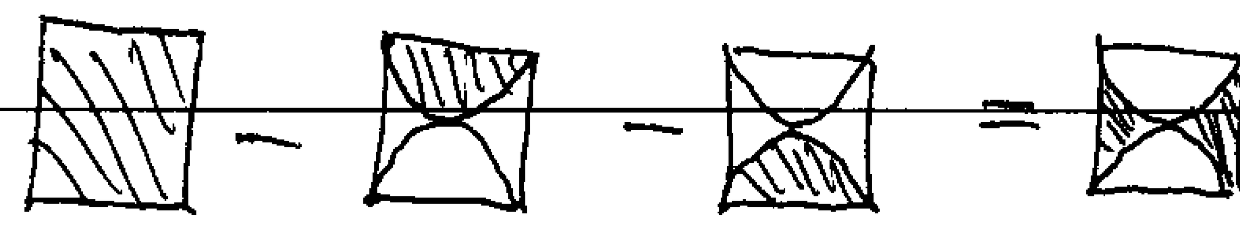
7.3-46

All of the three method could be used to find the volume.

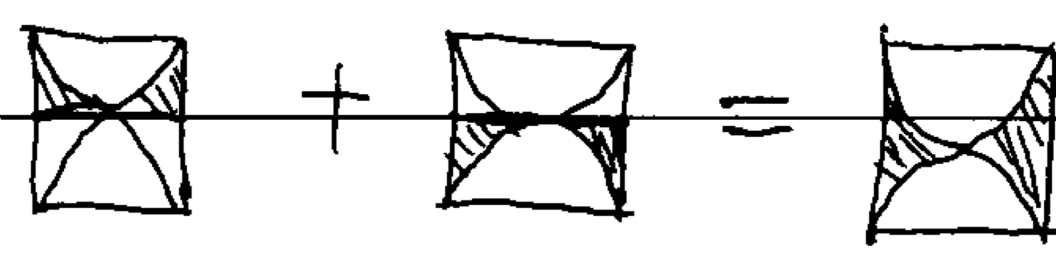
(i) disk:



(three integrals)

$$\int_{-1}^{1}\pi\cdot 1^2\,dy-\int_{0}^{1}\pi\cdot(\sqrt{y}-0)^2\,dy-\int_{-1}^{0}\pi\cdot(\sqrt[4]{-y}-0)^2\,dy=2\pi-\frac{\pi}{2}-\frac{2}{3}\pi=\frac{5}{6}\pi$$

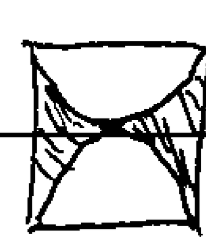
(ii) washer:



(two integrals)

$$\int_{0}^{1}\pi\left(1^2-(\sqrt{y}-0)^2\right)dy+\int_{-1}^{0}\pi\left(1^2-(\sqrt[4]{-y}-0)^2\right)dy=\frac{\pi}{2}+\frac{\pi}{3}=\frac{5}{6}\pi$$

(iii) Shell:



one integral

$$\int_{0}^{1}2\pi x\left(x^2-(-x^4)\right)dx=\int_{0}^{1}2\pi(x^3+x^5)\,dx=\frac{5}{6}\pi$$

7.4-46

The area swept out by AB

$$= \int_a^{a+h} 2\pi \sqrt{r^2-x^2} \sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\, dx \qquad \left(y=\sqrt{r^2-x^2} \Rightarrow \frac{dy}{dx} = \frac{-2x}{2\sqrt{r^2-x^2}} = \frac{x}{\sqrt{r^2-x^2}} \right)$$

$$= \int_a^{a+h} 2\pi \sqrt{(r^2-x^2)\left(1+\frac{x^2}{r^2-x^2}\right)}\, dx$$

$$= \int_a^{a+h} 2\pi \sqrt{r^2-x^2+x^2}\, dx = \int_a^{a+h} 2\pi r\, dx = 2\pi r \int_a^{a+h} 1\, dx$$

$$= 2\pi r [x]_a^{a+h} = \underbrace{2\pi r}_{\text{constant}} h .$$

Hence, the area does not depend on $\underbrace{\text{the location of the interval}}_{\Rightarrow a}$

but only depend on $\underbrace{\text{the length of the interval}}_{\Rightarrow h}$