Since $e > 2$

$$\Rightarrow \lim_{x\to\infty} \frac{e^x}{2^x} = \lim_{x\to\infty} \left(\frac{e}{2}\right)^x = \infty$$

and $\displaystyle\lim_{x\to\infty} \frac{x^2}{(\ln 2)^x} = \infty$, $(\ln 2 < 1)$

$$\lim_{x\to\infty} \frac{x^2}{2^x} = \lim_{x\to\infty} \frac{2x}{2^x \ln 2} = \lim_{x\to\infty} \frac{2}{2^x (\ln 2)^2} = 0$$

Hence from slowest growing to fastest as $x \to \infty$:

$$\Rightarrow (\ln 2)^x,\ x^2,\ 2^x,\ e^x$$

12.

(a) T (b) T (c) F (d) T (e) T (f) T (g) T (h) F

18.

Since $\displaystyle\lim_{n\to\infty} \frac{n}{\sqrt{n}\log_2 n} = \lim_{n\to\infty} \ln 2 \frac{\sqrt{n}}{\ln n} = \lim_{n\to\infty} \ln 2 \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}}$

$$= \lim_{n\to\infty} [\ln 2] \frac{\sqrt{n}}{2} = \infty$$

and $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}\log_2 n}{(\log_2 n)^2} = \lim_{n\to\infty} \frac{\sqrt{n}}{\log_2 n} = \lim_{n\to\infty} \ln 2 \frac{\sqrt{n}}{\ln n} = \lim_{n\to\infty} \ln 2 \frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n}}$

$$= \lim_{n\to\infty} [\ln 2] \frac{\sqrt{n}}{2} = \infty$$

Hence

n faster than $\sqrt{n}\log_2 n$ faster than $(\log_2 n)^2$.

$\Rightarrow$ The algorithm with steps order of $(\log_2 n)^2$ is the most efficient in the long run.

Problem 2 Show that $\frac{d\csc^{-1}y}{dy} = \frac{-1}{|y|\sqrt{y^2-1}}$, $|y|>1$

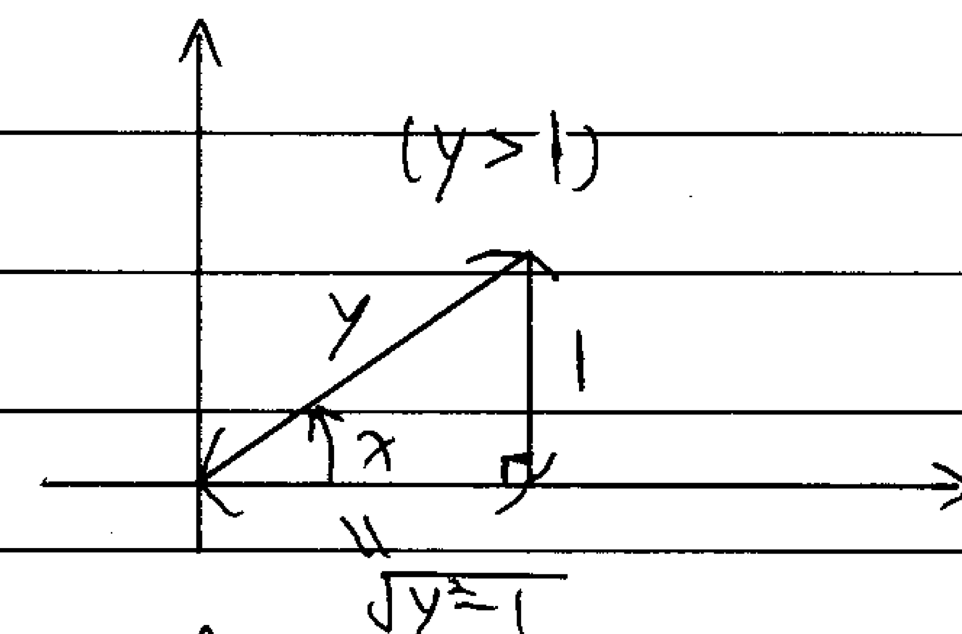
Pf:

Let $\csc^{-1}y = x \Rightarrow \csc x = y \Rightarrow \frac{d\csc x}{dy} = 1$

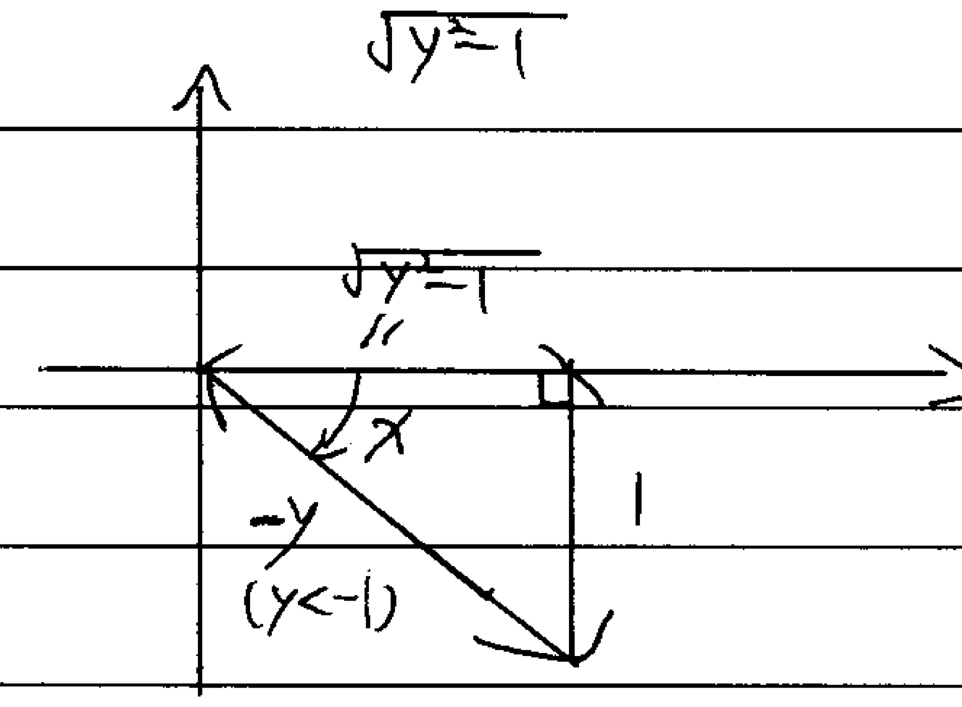
$\Rightarrow -\csc x \cdot \cot x \cdot \frac{dx}{dy} = 1$ ($\frac{dx}{dy}$ is what we want)

$\Rightarrow \frac{dx}{dy} = \frac{-1}{\csc x \cdot \cot x}$

$y>1 \Rightarrow \csc x = y$ and $\cot x = \sqrt{y^2-1}$



$y<-1 \Rightarrow \csc x = y$ and $\cot x = -\sqrt{y^2-1}$



Hence

$$\frac{d}{dy}(\csc^{-1}y) = \begin{cases} \frac{-1}{y\sqrt{y^2-1}}, & \text{if } y>1 \\ \frac{1}{y\sqrt{y^2-1}}, & \text{if } y<-1 \end{cases}$$

$$\Rightarrow \frac{d}{dy}(\csc^{-1}y) = \frac{-1}{|y|\sqrt{y^2-1}}, \quad |y|>1$$