6.3

32. $\frac{dy}{dt} = \frac{-\ln t}{t^2}$

34. $\frac{dy}{dx} = \frac{(1+\ln x)^2 - \ln x}{(1+\ln x)^2}$

36. $\frac{dy}{dt} = \frac{\cos t - \sin t}{\sin t}$

38. $\frac{dy}{d\theta} = \frac{1}{2\theta(1+\sqrt{\theta})}$

40. $\frac{dy}{d\theta} = \sec\theta$

86. $\frac{dy}{dx} = x^{(x+1)}\left(\frac{x+1}{x} + \ln x\right)$

88. $\frac{dy}{dt} = t^{\sqrt{t}}\left(\frac{\ln t}{2\sqrt{t}} + \frac{1}{\sqrt{t}}\right)$

90. $\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + (\cos x)\ln x\right)$

Problem 3

$$\lim_{n\to\infty} \sum_{k=\frac{n}{2}}^{n} \frac{1}{k} = \lim_{n\to\infty} \sum_{k=\frac{n}{2}}^{n} \frac{1}{\frac{k}{n}} \cdot \frac{1}{n}$$

$$= \int_{\frac{1}{2}}^{1} \frac{1}{x}\,dx = \ln|x|\Big]_{\frac{1}{2}}^{1} = 0 - \ln\frac{1}{2} = \ln 2$$

6.6

38 $\lim_{x\to 1^+} x^{\frac{1}{x-1}} = e$

40. $\lim_{x\to e^+} (\ln x)^{\frac{1}{x-e}} = e^{\frac{1}{e}}$

42. $\lim_{x\to\infty} x^{\frac{1}{\ln x}} = e$

44. $\lim_{x\to 0} (e^x + x)^{\frac{1}{x}} = e^2$

46. $\lim_{x\to 0^+} (1+\frac{1}{x})^x = 1$

54. $f(x) = \begin{cases} x+2, & x\neq 0 \\ 0, & x=0 \end{cases} \Rightarrow f'(x) = 1, \ x\neq 0$

$g(x) = \begin{cases} x+1, & x\neq 0 \\ 0, & x=0 \end{cases} \Rightarrow g'(x) = 1, \ x\neq 0$

Hence

$$\lim_{x\to 0} \frac{f'(x)}{g'(x)} = \frac{1}{1} = 1 \quad \text{and} \quad \lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{2}{1} = 2$$

This doesn't contradict l'Hôpital's rule.

Since f and g are not continuous at $x=0$

$\Rightarrow$ f and g are not differentiable at $x=0$

$\Rightarrow$ The result doesn't contradict l'Hôpital's rule.

55.

$$f(x) = \begin{cases} \dfrac{9x-3\sin 3x}{5x^3}, & x \neq 0 \\ c, & x=0. \end{cases}$$

$$\lim_{x\to 0} \frac{9x-3\sin 3x}{5x^3} = \lim_{x\to 0} \frac{9-9\cos 3x}{15x^2} = \lim_{x\to 0} \frac{27\sin 3x}{30x} = \lim_{x\to 0} \frac{27}{10}\,\frac{\sin 3x}{3x} = \frac{27}{10}$$

$\Rightarrow c = \frac{27}{10}$

Since $\lim_{x\to 0} f(x) = \frac{27}{10} = f(0)$ (if we let $c = \frac{27}{10}$)

$\Rightarrow$ f is continuous at $x=0$