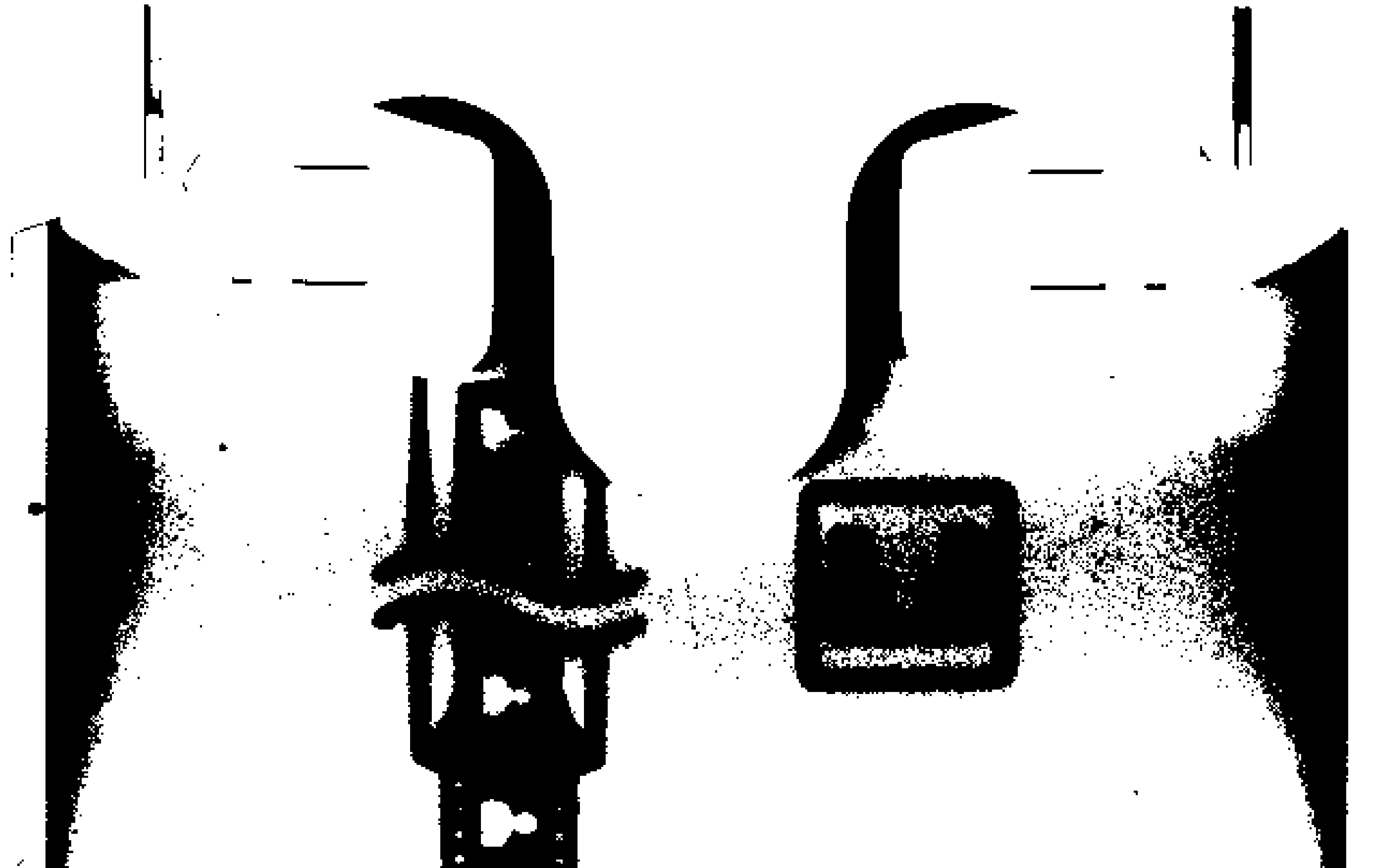
5.5-36

$$\int 2+\tan^2\theta \, d\theta = \int 2+(\sec^2\theta-1)\, d\theta = \int 1+\sec^2\theta \, d\theta$$

$$= \theta + \tan\theta + C$$



55. Differential equation: $\frac{d^3y}{dx^3} = 6 + \sin x$

Initial conditions: $\frac{d^2y}{dx^2} = -1$, $\frac{dy}{dx} = -5$ and $y = 2$ when $x = 0$

$\frac{d^3y}{dx^3} = 6 + \sin x \Rightarrow \frac{d^2y}{dx^2} = \int 6 + \sin x \, dx = 6x - \cos x + C$

$\frac{d^2y}{dx^2} = -1$, when $x=0 \Rightarrow 0 - 1 + C = -1 \Rightarrow C = 0 \Rightarrow \frac{d^2y}{dx^2} = 6x - \cos x$

$\Rightarrow \frac{dy}{dx} = \int 6x - \cos x \, dx = 3x^2 - \sin x + C$

$\frac{dy}{dx} = -5$ when $x=0 \Rightarrow 0 - 0 + C = -5 \Rightarrow C = -5 \Rightarrow \frac{dy}{dx} = 3x^2 - \sin x - 5$

$\Rightarrow y = \int 3x^2 - \sin x - 5 \, dx = x^3 + \cos x - 5x + C$

$y = 2$ when $x = 0 \Rightarrow 0 + 1 - 0 + C = 2 \Rightarrow C = 1 \Rightarrow y = x^3 + \cos x - 5x + 1$

Problem 2

Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \sin(\frac{k}{n})$

$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \sin(\frac{k}{n}) = \int_0^1 \sin x \, dx = (-\cos x)\Big|_0^1 = (-\cos 1) - (-\cos 0)$

$= 1 - \cos 1$

5.6-56 $\int_1^4 \frac{1}{2\sqrt{y}(1+\sqrt{y})^2}\,dy$ (let $z=1+\sqrt{y} \Rightarrow dz=\frac{1}{2\sqrt{y}}\,dy$)

$= \int_2^3 \frac{1}{z^2}\,dz = -z^{-1}\Big|_2^3 = \frac{1}{6}$.

58 $\int_{\pi}^{\frac{3\pi}{2}} \tan^{-5}(\frac{\theta}{6})\sec^2(\frac{\theta}{6})\,d\theta$ (let $y=\tan(\frac{\theta}{6}) \Rightarrow dy = \frac{1}{6}\sec^2(\frac{\theta}{6})\,d\theta$)

$= \int_{\frac{1}{\sqrt{3}}}^{1} \frac{6}{y^5}\,dy = -\frac{3}{2}y^{-4}\Big|_{\frac{1}{\sqrt{3}}}^{1} = 12$

67

a) $\frac{d(\sin^2 x + C_1)}{dx} = 2\sin x \cdot \cos x$

b) $\frac{d(-\cos^2 x + C_2)}{dx} = -2\cos x(-\sin x) = 2\sin x\cos x$

c) $\frac{d(-\frac{\cos 2x}{2} + C_3)}{dx} = \frac{\sin 2x}{2}\cdot 2 = \sin 2x = 2\sin x\cos x$

All of a), b) and c) are antiderivatives of $2\sin x\cos x$, all three integrations are correct

6.2-32

$f(x) = x^2-4x-5,\ x>2$

$\frac{df^{-1}}{dx}\Big|_{x=0} = \frac{1}{\frac{df}{dx}\big|_{x=5}} = \frac{1}{2x-4\big|_{x=5}} = \frac{1}{6}$

45

If f and g are one-to-one $\Rightarrow f\circ g$ is one-to-one

Since if $x \neq y \Rightarrow g(x) \neq g(y)$ (g is one-to-one)

$\Rightarrow f(g(x)) \neq f(g(y))$ (f is one-to-one)

46. If g is not one-to-one

$\Rightarrow \exists\, x \neq y$ s.t. $g(x) = g(y) \Rightarrow f(g(x)) = f(g(y))$

$\Rightarrow f \circ g$ is not one-to-one $(\rightarrow\leftarrow)$

Hence g must be one-to-one