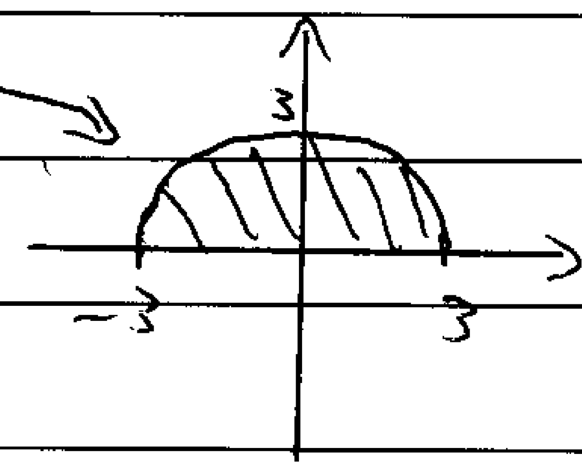
5.2-45

$y=\sqrt{9-x^2}$



$\int_{-3}^{3}\sqrt{9-x^2}\,dx$

$= \pi\cdot 3^2\cdot\frac{1}{2} = \frac{9}{2}\pi$

Problem 2

Evaluate $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2}$

Let $f(x)=\sqrt{1-x^2}$ on $[-1,1]$

$\begin{cases} c_k = \left(\frac{1-0}{n}\right)k = \frac{k}{n} \\ \Delta x_k = \frac{1-0}{n} = \frac{1}{n} \end{cases}$

$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{1-\left(\frac{k}{n}\right)^2} = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)\cdot\frac{1}{n}$ $\left(\lim_{\|P\|\to 0}\sum_{k=1}^{n} f(c_k)\Delta x_k = \int_a^b f(x)\,dx\right)$

$= \int_0^1 f(x)\,dx = \int_0^1\sqrt{1-x^2}\,dx$

(Similar to 5.2-45) $= \pi\cdot 1^2\cdot\frac{1}{4} = \frac{\pi}{4}$

5.3-36

$\frac{2}{5}\le\int_0^{0.5}\frac{1}{1+x^2}\,dx\le\frac{1}{2}$ and $\frac{1}{4}\le\int_{0.5}^{1}\frac{1}{1+x^2}\,dx\le\frac{2}{5}$

$\Rightarrow \frac{2}{5}+\frac{1}{4}\le\int_0^{0.5}\frac{1}{1+x^2}\,dx+\int_{0.5}^{1}\frac{1}{1+x^2}\,dx\le\frac{1}{2}+\frac{2}{5}$

$\Rightarrow \frac{13}{20}\le\int_0^1\frac{1}{1+x^2}\,dx\le\frac{9}{10}$

40

Let $h(x)=f(x)-g(x) \Rightarrow \frac{1}{b-a}\int_a^b h(x)\,dx = av(h) = h(c)$ for some $c\in[a,b]$

$\Rightarrow h(c) = \frac{1}{b-a}\int_a^b f(x)-g(x)\,dx = 0$ for some $c\in[a,b]$

$\Rightarrow f(c)=g(c)$ for some $c\in[a,b]$

Per-Duet

5.4-69 $\int_1^x f(t)dt = x^2-2x+1$

$f(x) = \frac{d}{dx}\left(\int_1^x f(t)dt\right) = \frac{d}{dx}(x^2-2x+1) = 2x-2$

73

a) f has a positive derivative for all values of x

$\Rightarrow f$ is continuous on $\mathbb{R}$

$\Rightarrow g$ is a differentiable function of x (Fundamental Theorem)

b)

By a) $\Rightarrow g$ is a continuous function of x

c)

$g'(x) = f(x) \Rightarrow g'(1) = f(1) = 0$

d), e), f)

$g''(x) = f'(x) \Rightarrow g''(1) = f'(1) > 0$ (g has no inflection at $x=1$)

$\Rightarrow g$ has local minimum at $x=1$

g)

Since $g'(1)=0$ & $g''(x) = f'(x) > 0 \Rightarrow g'$ is increasing on $\mathbb{R}$

Hence $\frac{dg}{dx} = g'(x)$ crosses the x-axis at $x=1$