4.5-26

$V(\text{volume}) = \pi x^2 \cdot (y+3)/3$ and $x^2+y^2=9$

$= \frac{\pi}{3}(9-y^2)(y+3)$

$\Rightarrow \frac{dV}{dy} = \frac{\pi}{3}\left(-2y(y+3) + (9-y^2)\right) = \frac{\pi}{3}(-3y^2-6y+9) = -\pi(y^2+2y-3)$

$= -\pi(y-1)(y+3)$

Hence when $y=1$, we have largest right circular cone

$(y-1)(y-(-3))$	$+$	$-$	$+$
$\frac{dV}{dy}$	$-$	$+$	$-$
		-3	1

Volume $= \frac{\pi}{3}\cdot 8\cdot 4 = \frac{32}{3}\pi$

36. $f(x)=3+4\cos x+\cos 2x$

(a) Since $\cos x$ has period 2π and $\cos 2x$ has period π

$\Rightarrow f$ has period 2π

(b)

$f(x)=3+4\cos x+\cos 2x$

$\Rightarrow f'(x)=-4\sin x-2\sin 2x=-4\sin x-4\sin x\cos x=-4\sin x(1+\cos x)$

If $0=f'(x) \Rightarrow \sin x(1+\cos x)=0$

$\Rightarrow \sin x=0$ or $\cos x=-1$

$\Rightarrow x=0,\pi,2\pi$

	0 to π	π to 2π
$\sin x$	$+$	$-$
$1+\cos x$	$+$	$+$
f'	$-$	$+$

Hence f has a local minimum

at $x=\pi \Rightarrow f(\pi)=3-4+1=0$

$\Rightarrow f\geq 0$

4.6-23(b)

Let $f(x)=x^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$

$\Rightarrow f'(x)=nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\cdots+a_1$

If $\exists$ a,b s.t. $f(a)=0=f(b)$, then by Rolle's Theorem,

there exists $c\in(a,b)$ s.t. $f'(c)=0$ #

25. If $f(0)=f(1)$

then by Mean Value Theorem, there exist $c\in(0,1)$ such that

$f'(c)=\frac{f(1)-f(0)}{1-0}=\frac{0}{1}=0$ $(\rightarrow\leftarrow)$

Hence $f(0)\neq f(1)$

26.

W.L.O.G. may assume $a<b$, since $\sin x$ is differentiable on $\mathbb{R}$,

by Mean Value Theorem, $\exists\, c\in(a,b)$ such that

$$\cos c=\frac{\sin b-\sin a}{b-a}\Rightarrow|\sin b-\sin a|=|b-a||\cos c|$$

$$(\text{Here, } |\cos c|\le 1)\qquad\le|b-a|$$

28.

Let $h(x)=f(x)-g(x)$ is differentiable on $[a,b]$

$\Rightarrow h(a)=h(b)=0$

By Rolle's Theorem, $\exists\, c\in(a,b)$ s.t. $h'(c)=0$

$\Rightarrow f'(c)-g'(c)=0\Rightarrow f'(c)=g'(c)$ for some $c\in(a,b)$

33.

$f'(x)=2$ for all $x\Rightarrow\forall\, a,b\in\mathbb{R}$, $\frac{f(b)-f(a)}{b-a}=f'(c)=2$, for some $c\in(a,b)$

slope

$\Rightarrow f$ is a line function and $f(0)=5$

$\Rightarrow f(x)=5+2x$