Hw 07

4.4-41 $f(x)$ and $g(x)$ are polynomials in x and

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 2$$

$\Rightarrow$ If $g(x) = ax^m + (\text{degree} < m)$, then $f(x) = 2ax^m + (\text{degree} < m)$

where $m \in \mathbb{N} \cup \{0\}$, $a \in \mathbb{R} \setminus \{0\}$

$$\Rightarrow \lim_{x\to-\infty} \frac{f(x)}{g(x)} = \lim_{x\to-\infty} \frac{2a + \frac{(\text{degree}<m)}{x^m}}{a + \frac{(\text{degree}<m)}{x^m}} = \frac{2a}{a} = 2$$

58

Graph $y = x^{\frac{2}{3}}/(x^2-1)$

$\Rightarrow$ Vertical asymptotes : $x=1$, $x=-1$

$$y' = -\frac{2}{3}\frac{2x^2+1}{x^{\frac{1}{3}}(x^2-1)^2}$$

$\Rightarrow$ Critical points : $x=0, 1, -1$

$$y'' = \frac{2}{9}\frac{14x^4+23x^2-1}{x^{\frac{4}{3}}(x^2-1)^3} = \frac{2}{9}\frac{\left(x^2-\frac{-23-\sqrt{585}}{28}\right)\left(x-\sqrt{\frac{-23+\sqrt{585}}{28}}\right)\left(x+\sqrt{\frac{-23+\sqrt{585}}{28}}\right)}{x^{\frac{4}{3}}(x^2-1)^3}$$

$\Rightarrow$ Inflection points : $x=0, 1, -1, \sqrt{\frac{-23+\sqrt{585}}{28}}, -\sqrt{\frac{-23+\sqrt{585}}{28}}$, where $\sqrt{\frac{-23+\sqrt{585}}{28}} \approx 0.206$

	①	②	③	④	⑤	⑥
$x^{\frac{1}{3}}$	−	−	−	+	+	+
y'	+	+	+	−	−	−
$\left(x^2-\frac{-23-\sqrt{585}}{28}\right)$	+	+	+	+	+	+
$\left(x^2-\frac{-23+\sqrt{585}}{28}\right)$	+	+	−	−	+	+
$x^{\frac{4}{3}}$	+	+	+	+	+	+
$(x^2-1)^3$	+	−	−	−	−	+
y''	+	−	+	+	−	+

①: $-2 \le x < -1$

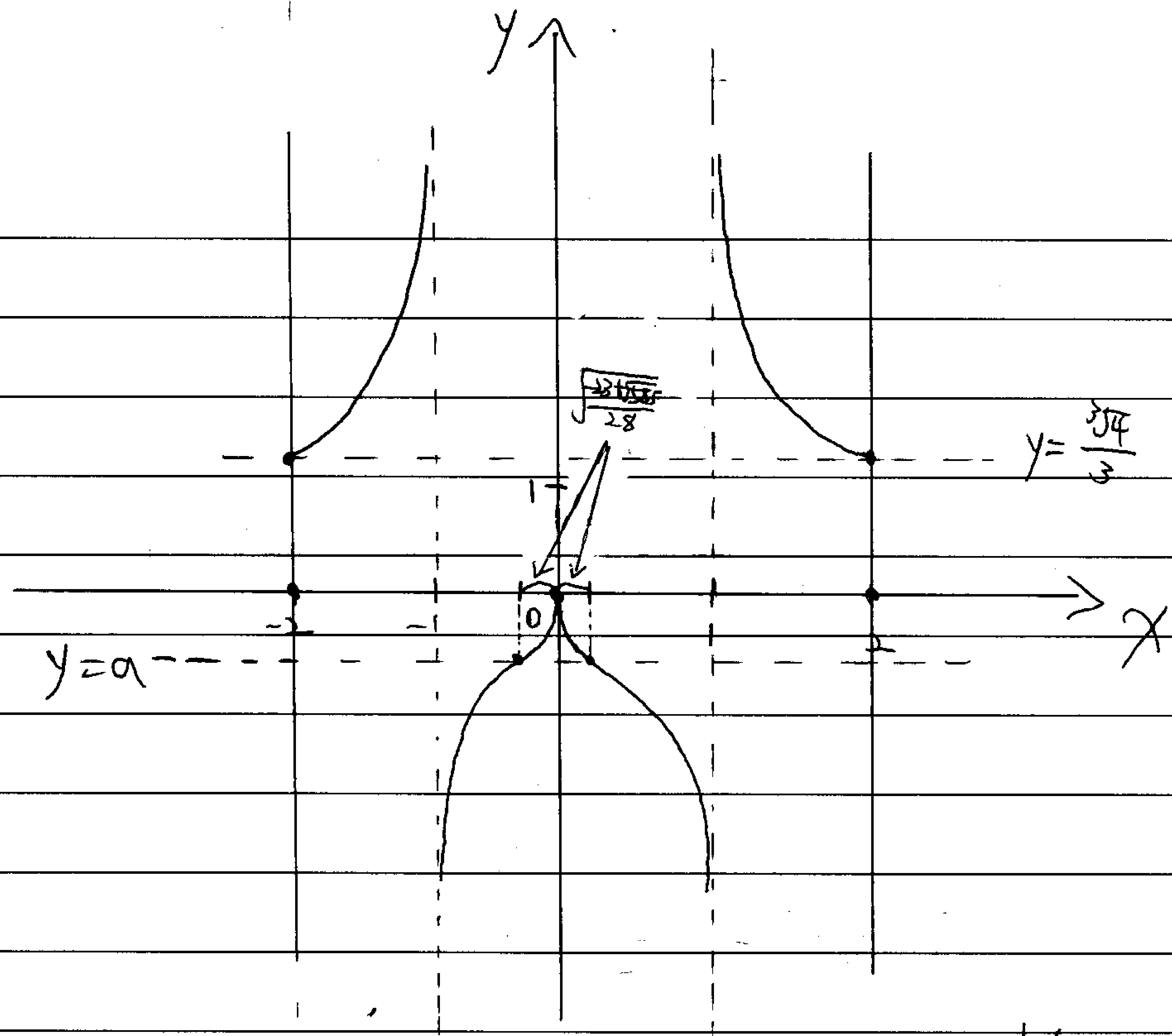
②: $-1 < x < -\sqrt{\frac{-23+\sqrt{585}}{28}}$

③: $-\sqrt{\frac{-23+\sqrt{585}}{28}} < x < 0$

④: $0 < x < \sqrt{\frac{-23+\sqrt{585}}{28}}$

⑤: $\sqrt{\frac{-23+\sqrt{585}}{28}} < x < 1$

⑥: $1 < x \le 2$



$x=0,\ y=0 \Rightarrow (0,0)$, $x=-\sqrt{\frac{-23+\sqrt{585}}{28}}$, $y=\frac{x^{\frac{2}{3}}}{x^2-1} \overset{\text{let}}{=} a<0 \Rightarrow \left(-\sqrt{\frac{-23+\sqrt{585}}{28}},\ a\right)$

$x=\sqrt{\frac{-23+\sqrt{585}}{28}},\ y=a \Rightarrow \left(\sqrt{\frac{-23+\sqrt{585}}{28}},\ a\right)$, $x=-2,\ y=\frac{\sqrt[3]{4}}{3} \Rightarrow \left(-2,\ \frac{\sqrt[3]{4}}{3}\right)$

$x=2,\ y=\frac{\sqrt[3]{4}}{3} \Rightarrow \left(2,\ \frac{\sqrt[3]{4}}{3}\right)$