4.2-31.

a) $f'(x) = (x-1)(x+2)(x-3)$

$f'(x) = 0 \Rightarrow x=1$ or $x=-2$ or $x=3$

$\Rightarrow x = 1, -2, 3$ are critical points of f

b)

$x<-2$	$-2<x<1$	$1<x<3$	$x>3$
$f'(x)<0$	$f'(x)>0$	$f'(x)<0$	$f'(x)>0$

$\Rightarrow \begin{cases} f \text{ is increasing on } (-2,1) \cup (3,\infty) \\ f \text{ is decreasing on } (-\infty,-2) \cup (1,3) \end{cases}$

c)

According to the same table in b)

$\Rightarrow f$ have local minimum at $x = -2, 3$

& local maximum at $x = 1$

33

a) $f'(x) = x^{-\frac{1}{3}}(x+2)$

$f'(x) = 0 \Rightarrow x = -2$, $f'(x)$ does not exist at $x=0$

$\Rightarrow x = -2, 0$ are critical points of f

b)

$\begin{cases} f \text{ is increasing on } (-\infty,-2) \cup (0,\infty) \\ f \text{ is decreasing on } (-2,0) \end{cases} \Leftarrow$

$x<-2$	$-2<x<0$	$x>0$
$f'(x)>0$	$f'(x)<0$	$f'(x)>0$

c) See text book's solution

57 $g(x) = x^2 - 4x + 4 \,, \quad 1 \le x < \infty$

$\Rightarrow g'(x) = 2x - 4 \,, \quad 1 \le x < \infty$

$g'(x) = 0 \Rightarrow x = 2$ (critical point) $\Rightarrow \begin{cases} g'(x) < 0 \,, & x < 2 \\ g'(x) > 0 \,, & x > 2 \end{cases}$

$\Rightarrow g(x)$ has local minimum $g(2) = 0$ at $x = 2$

endpoints $\Rightarrow g(1) = 1$ & $g'(x) < 0 \,, \quad 1 \le x < 2$

$\Rightarrow g(x)$ has local maximum 1 at $x = 1$

61 $h(x) = \frac{x^3}{3} - 2x^2 + 4x \,, \quad 0 \le x < \infty$

$\Rightarrow h'(x) = x^2 - 4x + 4 = (x-2)^2 \,, \quad 0 \le x < \infty$

$h'(x) = 0 \Rightarrow x = 2$ (critical point) $\Rightarrow h'(x) \ge 0 \,, \quad x \ge 0$

$\Rightarrow h(x)$ has no extreme value in $(0, \infty)$

endpoint $\Rightarrow h(0) = 0$ & $h'(x) > 0 \,, \quad x \ge 0$

$\Rightarrow h(x)$ has local minimum 0 at $x = 0$

63 $y = x^3 - 6x \quad , -2 \le x \le 2$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 6 = 3(x^2-2) \quad , -2 \le x \le 2$

$\frac{dy}{dx} = 0 \Rightarrow x = \sqrt{2}, -\sqrt{2}$ (critical points)

$\Rightarrow$

$-2 < x < -\sqrt{2}$	$-\sqrt{2} < x < \sqrt{2}$	$\sqrt{2} < x < 2$
$\frac{dy}{dx} > 0$	$\frac{dy}{dx} < 0$	$\frac{dy}{dx} > 0$

$\Rightarrow y = x^3 - 6x$ has local maximum $4\sqrt{2}$ at $x = -\sqrt{2}$

& local minimum $-4\sqrt{2}$ at $x = \sqrt{2}$

endpoints: $y(-2) = 4$ & $\frac{dy}{dx} > 0 \quad , -2 < x < -\sqrt{2}$

$\Rightarrow y$ has local minimum 4 at $x = -2$

$y(2) = -4$ & $\frac{dy}{dx} > 0 \quad , \sqrt{2} < x < 2$

$\Rightarrow y$ has local maximum -4 at $x = 2$

81

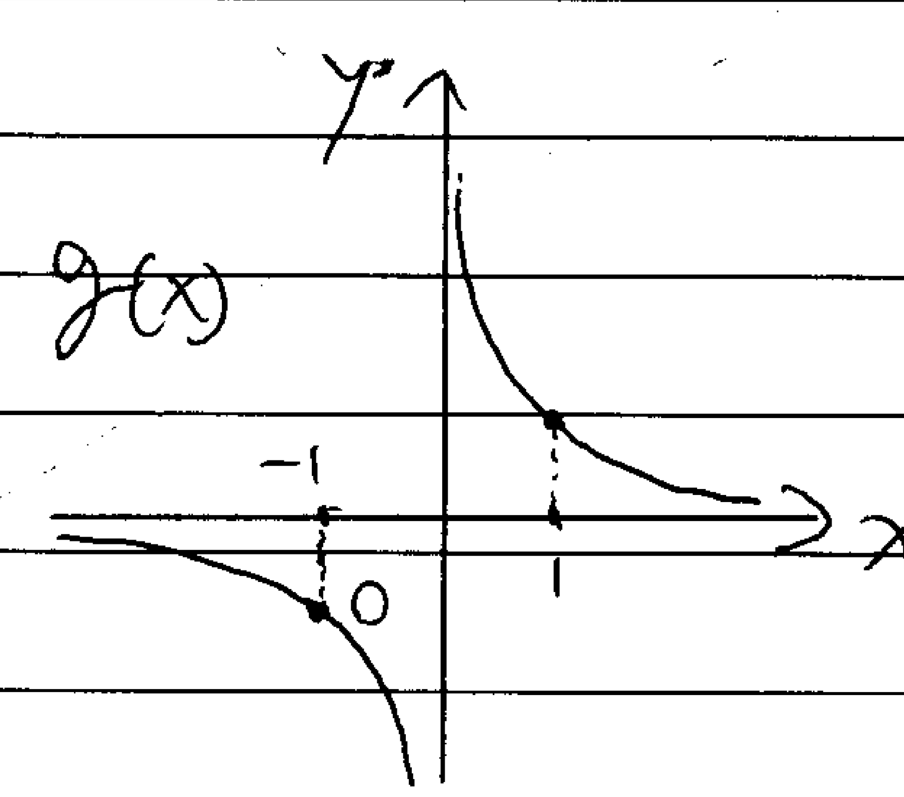
a) $g(x) = \frac{1}{x} \Rightarrow g'(x) = -\frac{1}{x^2} < 0 \quad , x \neq 0$

$\Rightarrow g$ is decreasing at $x \neq 0$

b)

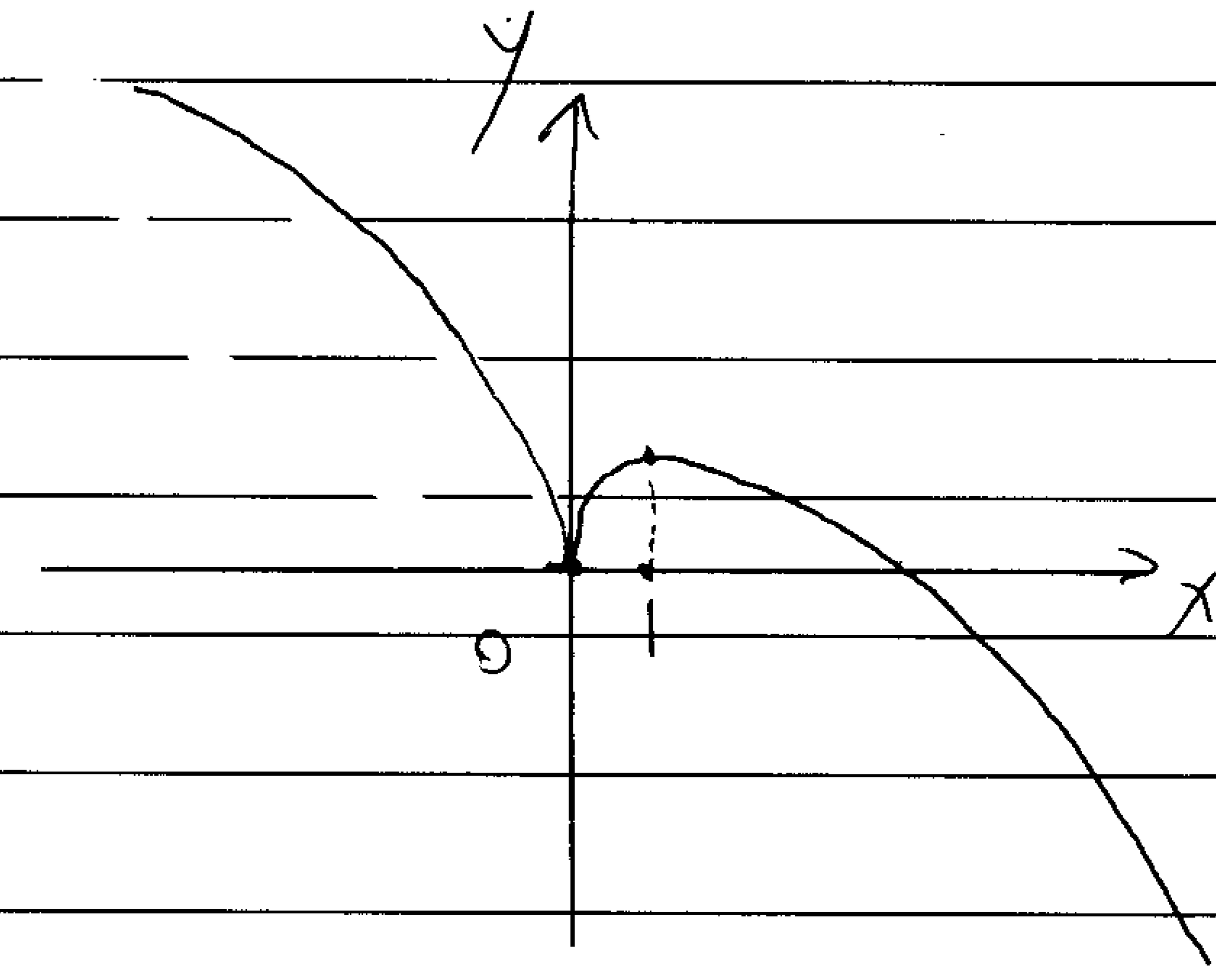
g is decreasing on $(-\infty, 0) \cup (0, \infty)$

but g is not continuous at $x = 0$



4.3-34

$y = 5x^{\frac{2}{5}} - 2x$



$\Rightarrow \frac{dy}{dx} = 2x^{-\frac{3}{5}} - 2$ & $\frac{d^2y}{dx^2} = -\frac{6}{5}x^{-\frac{8}{5}}$

$\frac{dy}{dx} = 0 \Rightarrow x = 1$ & $\left.\frac{d^2y}{dx^2}\right|_{x=1} = -\frac{6}{5} < 0$

$\Rightarrow$ y has local maximum at $x=1$

Moreover, y has no derivative at $x=0$

$\Rightarrow$

	$x<0$	$0<x<1$	$x>1$
	$\frac{dy}{dx}<0$	$\frac{dy}{dx}>0$	$\frac{dy}{dx}<0$
(correction) $\Rightarrow$	$\frac{d^2y}{dx^2}<0$	$\frac{d^2y}{dx^2}<0$	$\frac{d^2y}{dx^2}<0$

$\Rightarrow$ y has local minimum at $x=0$

75

$y = x^3 + bx^2 + cx + d$ have a point of inflection at $x=1$

$\Rightarrow \left.\frac{d^2y}{dx^2}\right|_{x=1} = 0$

$\frac{d^2y}{dx^2} = 6x + 2b \overset{x=1}{\Rightarrow} 6 + 2b = 0 \Rightarrow b = -3$

78.

No!

$y = x^4$ has no inflection point at the origin,

even though $\frac{d^2y}{dx^2} = 0$ at $x=0$

80

$y = ax^3 + bx^2 + cx + d$, $a \neq 0$

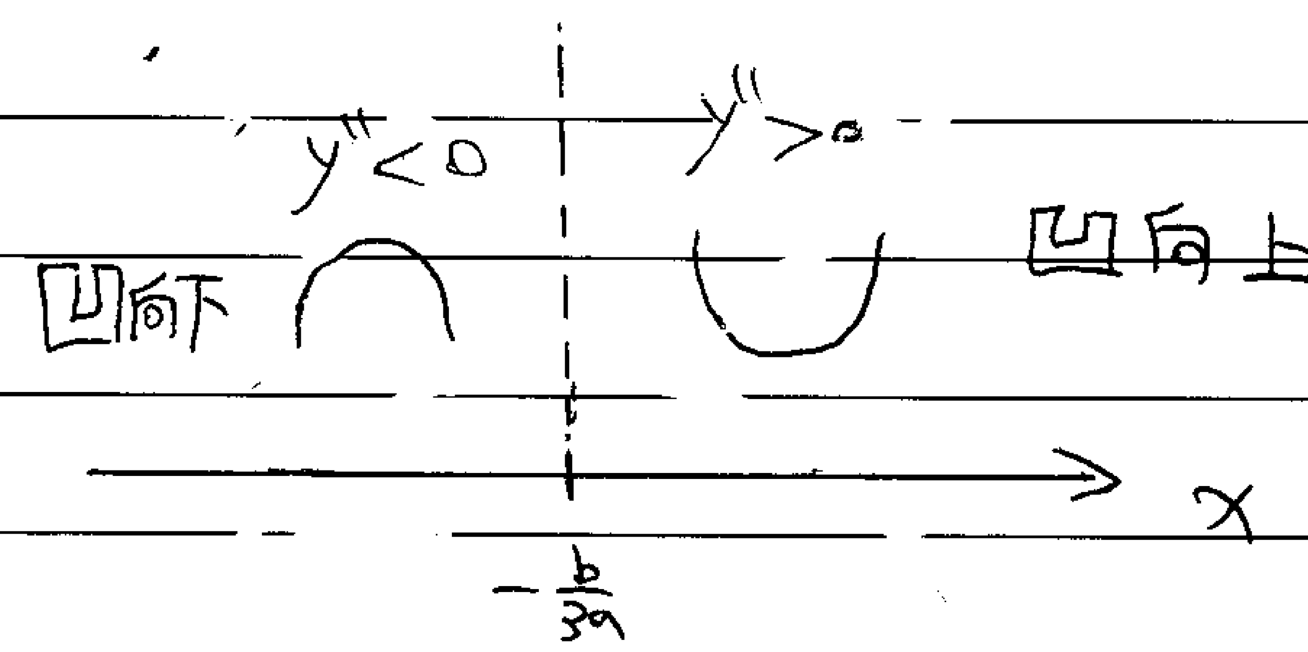
$\Rightarrow \frac{d^2y}{dx^2} = 6ax + 2b$

$\frac{d^2y}{dx^2} = 0 \Rightarrow x = -\frac{b}{3a}$

(i) If $a > 0 \Rightarrow \frac{d^2y}{dx^2} > 0$, $x > -\frac{b}{3a}$

$\frac{d^2y}{dx^2} < 0$, $x < -\frac{b}{3a}$

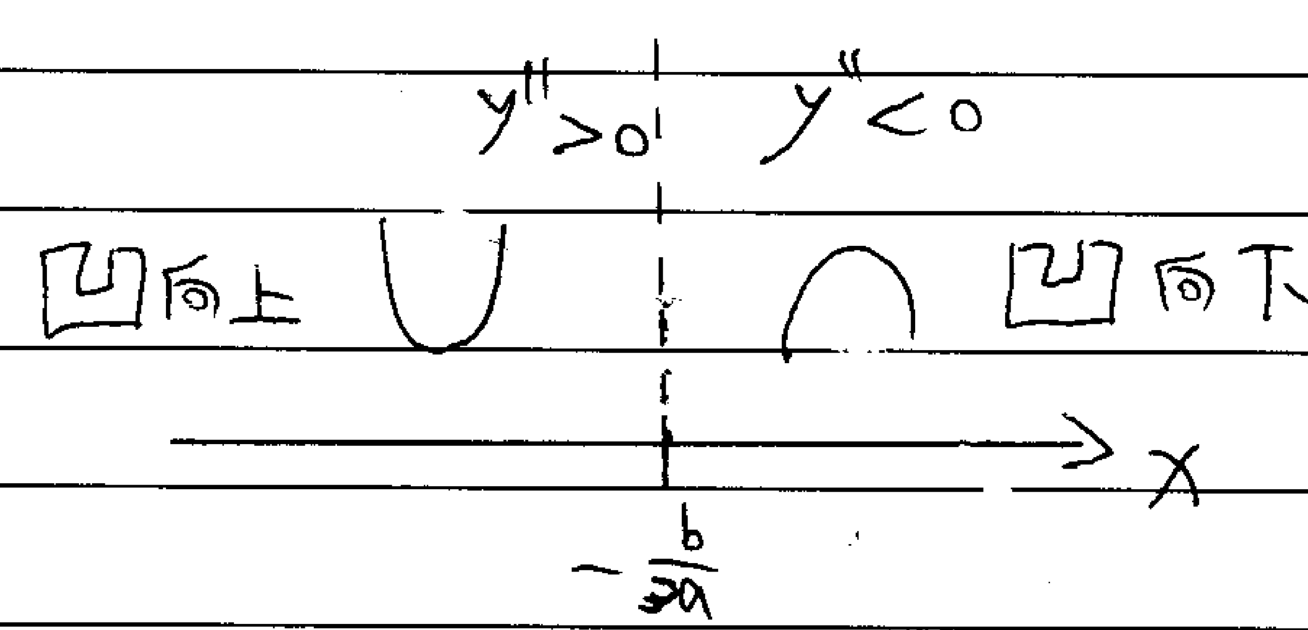
$\Rightarrow$



(ii) If $a < 0 \Rightarrow \frac{d^2y}{dx^2} < 0$, $x > -\frac{b}{3a}$

$\frac{d^2y}{dx^2} > 0$, $x < -\frac{b}{3a}$

$\Rightarrow$



Per-Duet