3.5 52. $\frac{dr}{d\theta} = 2\theta\cos\frac{1}{\theta} + \sin\frac{1}{\theta}$ 54. $\frac{dy}{dt} = 2\pi\left[\sec^2(\pi t)\right]\cdot\tan(\pi t)$

56. $\frac{dy}{dt} = \left(1+\cot\left(\frac{t}{2}\right)\right)^{-3}\cdot\csc^2\left(\frac{t}{2}\right)$ 58. $\frac{dy}{dt} = \left[\cos(\sin t)\right]\cdot\cos t$

60. $\frac{dy}{dt} = -\frac{5}{3}\left[\sin\left(5\sin\left(\frac{t}{3}\right)\right)\right]\cdot\cos\left(\frac{t}{3}\right)$ 62. $\frac{dy}{dt} = -7\left(1+\cos^2(7t)\right)^2\left[\cos(7t)\right]\cdot\sin(7t)$

64. $\frac{d^2y}{dx^2} = 2\left[\sec^2\left(\frac{x}{3}\right)\right]\tan\left(\frac{x}{3}\right)$

66. $(f\circ g)'(-1)=1$ 68. $(f\circ g)'\left(\frac{1}{4}\right)=5\pi$ 70. $(f\circ g)'(-1)=-8$

3.6-58

$\frac{ds}{dt} = v = k\sqrt{s}$

acceleration (加速度) is denoted by "a"

$a = \frac{d^2s}{dt^2}$

$= \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d}{dt}\left(k\sqrt{s}\right) = k\frac{1}{2\sqrt{s}}\cdot\frac{ds}{dt}$ ⇐ chain rule

$= \frac{k}{2\sqrt{s}}\cdot k\sqrt{s} = \frac{k^2}{2}$

⇒ acceleration is a constant

3.7-10 $f(x) = 1+x, \quad x_0 = 8.1$

$\Rightarrow f(x) \approx L(x) = f(8) + f'(8)(x-8) = 9 + 1\cdot(x-8) = x+1$

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$(1+x)^k \approx 1+kx$

a) $(1.0002)^{50} \approx 1 + 50 \cdot 0.0002 = 1 + 0.01 = 1.01$

b) $\sqrt[3]{1.009} = (1+0.009)^{\frac{1}{3}} \approx 1 + \frac{1}{3} \cdot 0.009 = 1.003$

Use $f(x) - L(x) = \frac{1}{2} f''(c)(x-x_0)^2$ & the error bound in Problem 3

$\Rightarrow \sqrt[3]{1.009} - 1.003 = \frac{1}{2}\left[\frac{1}{3}\cdot\left(-\frac{2}{3}\right)\cdot(1+c)^{\frac{1}{3}-2}\right](0.009-0)^2$ where c lies between 0 & 0.009

$\Rightarrow \left|\sqrt[3]{1.009} - 1.003\right| \le \frac{1}{9}\cdot(1+0)^{-\frac{5}{3}}\cdot 0.009^2 = 9\times10^{-6}$

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W: weight, r: radius, m: density (constant), h: high

$W = \pi r^2 \cdot h \cdot m \Rightarrow dW = 2\pi r h m dr \Rightarrow \frac{dW}{W} = \frac{2\pi r h m dr}{\pi r^2 h m} = 2\frac{dr}{r}$

$\frac{dW}{W} = \frac{1}{1000} = 0.1\,\% \Rightarrow \frac{dr}{r} = \frac{1}{2}\frac{dW}{W} = 0.05\,\%$

51. f(x) has a horizontal tangent at $x=a \Rightarrow f'(a) = 0$

$\Rightarrow L(x) = f(a) + f'(a)(x-a) = f(a)$: constant function

Ch3-90 $f(x)=\frac{1}{1+\tan x} \Rightarrow f'(x)=-\frac{\sec^2 x}{(1+\tan x)^2}$

$\Rightarrow L(x)=f(0)+f'(0)(x-0)=1+(-1)x=1-x$

"For $\frac{1}{1+\sin(2x)}$"

$\frac{1}{1+x}\approx 1-x$ & $\sin 2x \approx 0+2\cdot(x-0)=2x$

$\Rightarrow \frac{1}{1+\sin(2x)}\approx 1-2x$

Check! $g(x)=\frac{1}{1+\sin 2x} \Rightarrow g'(x)=-\frac{2\cos 2x}{(1+\sin 2x)^2}$

$\Rightarrow L(x)=1+(-2)\cdot(x-0)=1-2x$