3.2-57

(a) $y = f(x) = x^3 - 4x + 1 \Rightarrow f'(x) = 3x^2 - 4$

$f'(2) = 8 \Rightarrow$ Slope of the equation perpendicular to the tangent to the curve : $-\frac{1}{8}$

Equation : $y = -\frac{1}{8}(x-2) + 1 = -\frac{1}{8}x + \frac{5}{4}$

(b)

$f''(x) = 6x$, solve $6x = 0 \Rightarrow x = 0$

Moreover , $\underbrace{f''(x) < 0 \, , \, x < 0}_{\text{decreasing}}$ & $\underbrace{f''(x) > 0 \, , \, x > 0}_{\text{increasing}}$

Hence we have the smallest slope $f'(0) = -4$ at $(0, 1)$

3.2-57 (c) Solve $f'(x)=3x^2-4=8$

$\Rightarrow 3x^2=12 \Rightarrow x^2=4 \Rightarrow x=\pm 2$

$f(2)=1 \Rightarrow y=8(x-2)+1=8x-15$

$f(-2)=1 \Rightarrow y=8(x+2)+1=8x+17$

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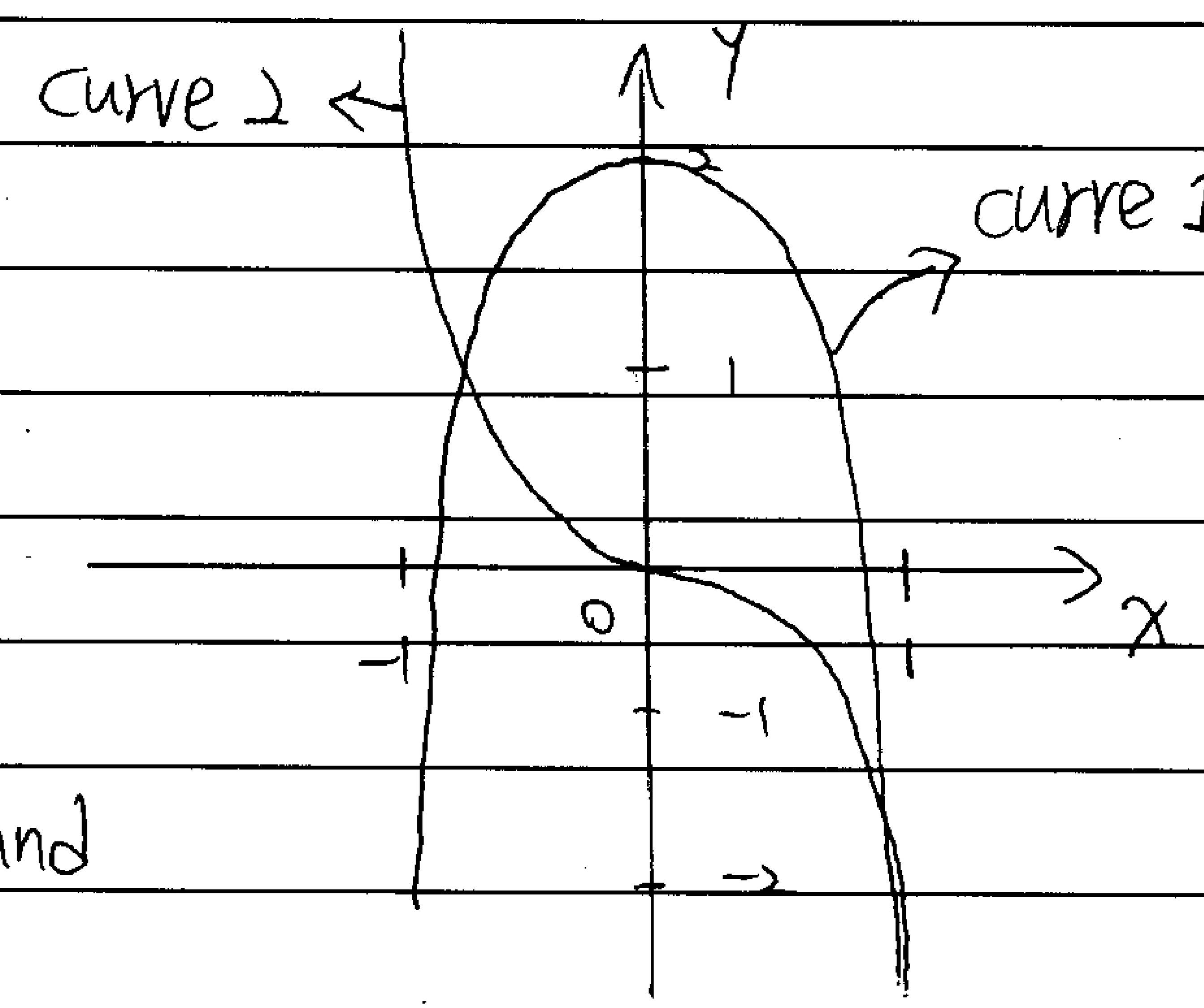

$f(x)$ is curve 1

$f'(x)$ is curve 2

Since all the slope of the tangent line to curve 2 is negative, and there are positive values in curve 1.

So curve 1 can not be the derivative of curve 2

Consider the slope of curve 1, we can find out that the slope decreasing on $(-1, 1)$ and have 0 derivative at $x=0$, curve 2 satisfy all the phenomenon we found in curve 1

3.4-53 Find $\frac{d^{999}}{dx^{999}}(\cos x)$

$\frac{d\cos x}{dx} = -\sin x$, $\frac{d^2\cos x}{dx^2} = -\cos x$, $\frac{d^3\cos x}{dx^3} = \sin x$, $\frac{d^4\cos x}{dx^4} = \cos x$

and $999 \div 4 = 249 \cdots 3$

Hence $\frac{d^{999}}{dx^{999}}(\cos x) = \sin x$

Problem 5

$$\frac{d}{dx}(f_1(x)f_2(x)\cdots f_n(x)) = f_1'(x)\cdot f_2(x)\cdots f_n(x) + f_1(x)\cdot(f_2(x)\cdots f_n(x))'$$

$$= f_1'(x)\cdot f_2(x)\cdots f_n(x) + f_1(x)\cdot f_2'(x)\cdot f_3(x)\cdots f_n(x) + f_1(x)\cdot f_2(x)\cdot(f_3(x)\cdots f_n(x))'$$

$$= \cdots$$

$$= f_1'(x)\cdot f_2(x)\cdots f_n(x) + f_1(x)\cdot f_2'(x)\cdot f_3(x)\cdots f_n(x) + \cdots$$

$$+ \underbrace{f_1(x)\cdots f_{i-1}(x)\cdot f_i'(x)\cdot f_{i+1}(x)\cdots f_n(x)}_{\text{the } i\text{th term}} + \cdots + f_1(x)\cdots f_{n-1}(x)\cdot f_n'(x)$$

6.

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) \\ h(x) & k(x) \end{vmatrix} = \frac{d}{dx}(f(x)k(x) - g(x)h(x))$$

$$= f'(x)k(x) + f(x)k'(x) - (g'(x)h(x) + g(x)h'(x))$$

$$= (f'(x)k(x) - g(x)h'(x)) + (f(x)k'(x) - g'(x)h(x))$$

$$= \begin{vmatrix} f'(x) & g(x) \\ h'(x) & k(x) \end{vmatrix} + \begin{vmatrix} f(x) & g'(x) \\ h(x) & k'(x) \end{vmatrix}$$

The other equality is similar

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & n(x) \end{vmatrix} = \frac{d}{dx}\left(f(x)\begin{vmatrix} j(x) & k(x) \\ m(x) & n(x) \end{vmatrix} - i(x)\begin{vmatrix} g(x) & h(x) \\ m(x) & n(x) \end{vmatrix} + l(x)\begin{vmatrix} g(x) & h(x) \\ j(x) & k(x) \end{vmatrix} \right)$$

$$= \underbrace{f'(x)\begin{vmatrix} j(x) & k(x) \\ m(x) & n(x) \end{vmatrix}}_{1} + f(x)\left(\underbrace{\begin{vmatrix} j'(x) & k(x) \\ m'(x) & n(x) \end{vmatrix}}_{2} + \underbrace{\begin{vmatrix} j(x) & k'(x) \\ m(x) & n'(x) \end{vmatrix}}_{3} \right) - \underbrace{i'(x)\begin{vmatrix} g(x) & h(x) \\ m(x) & n(x) \end{vmatrix}}_{1}$$

$$- i(x)\left(\underbrace{\begin{vmatrix} g'(x) & h(x) \\ m'(x) & n(x) \end{vmatrix}}_{2} + \underbrace{\begin{vmatrix} g(x) & h'(x) \\ m(x) & n'(x) \end{vmatrix}}_{3} \right) + \underbrace{l'(x)\begin{vmatrix} g(x) & h(x) \\ j(x) & k(x) \end{vmatrix}}_{1} + l(x)\left(\underbrace{\begin{vmatrix} g'(x) & h(x) \\ j'(x) & k(x) \end{vmatrix}}_{2} + \underbrace{\begin{vmatrix} g(x) & h'(x) \\ j(x) & k'(x) \end{vmatrix}}_{3} \right)$$

$$= \underbrace{\begin{vmatrix} f(x) & g(x) & h(x) \\ i'(x) & j(x) & k(x) \\ l'(x) & m(x) & n(x) \end{vmatrix}}_{1} + \underbrace{\begin{vmatrix} f(x) & g'(x) & h(x) \\ i(x) & j'(x) & k(x) \\ l(x) & m'(x) & h(x) \end{vmatrix}}_{2} + \underbrace{\begin{vmatrix} f(x) & g(x) & h'(x) \\ i(x) & j(x) & k'(x) \\ l(x) & m(x) & n'(x) \end{vmatrix}}_{3}$$

Similarly, we have

$$\frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & n(x) \end{vmatrix} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ i(x) & j(x) & k(x) \\ l(x) & m(x) & n(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ i'(x) & j'(x) & k'(x) \\ l(x) & m(x) & n(x) \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) \\ l'(x) & m'(x) & n'(x) \end{vmatrix}$$

And the 4X4

$$\frac{d}{dx}\begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} = \begin{vmatrix} a'(x) & b'(x) & c'(x) & d'(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} + \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e'(x) & f'(x) & g'(x) & h'(x) \\ i(x) & j(x) & k(x) & l(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix}$$

$$+ \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i'(x) & j'(x) & k'(x) & l'(x) \\ m(x) & n(x) & p(x) & q(x) \end{vmatrix} + \begin{vmatrix} a(x) & b(x) & c(x) & d(x) \\ e(x) & f(x) & g(x) & h(x) \\ i(x) & j(x) & k(x) & l(x) \\ m'(x) & n'(x) & p'(x) & q'(x) \end{vmatrix}$$