

Homework Assignment for Week 15

Assigned Jun 04, 2009.

1. Section 15.5: Problems 3, 15, 19, 21, 33, 35.
2. Section 15.6: Problems 1, 15, 17, 19, 25, 27, 29, 37.
3. Section 15.7: Problems 1, 3, 7, 10, 13(a,c), 15, 16, 19.
4. This exercise is to show that Flux, Circulation and the Curl of a vector field does not depend on the coordinate you choose.

Let x', y' be the coordinate axis obtained by rotating the x, y axis by a fixed angle θ .

- (a) Express x', y' in terms of x, y and vice versa.
- (b) Express $\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}$ in terms of $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$ and vice versa.
- (c) Let (M, N) be the components of a vector field \mathbf{F} in the original (x, y) coordinate. Express the components of \mathbf{F} , (M', N') in the new (x', y') coordinates in terms of M and N .
- (d) Use chain rule to verify that

$$\frac{\partial N'}{\partial x'} - \frac{\partial M'}{\partial y'} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

and

$$\frac{\partial M'}{\partial x'} + \frac{\partial N'}{\partial y'} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

- (e) Express the unit vectors \hat{x}', \hat{y}' in terms of \hat{x}, \hat{y} and vice versa.
- (f) Let x', y' be defined as above. In 3D, we perform the change of variable from (x, y, z) to (x', y', z) (z coordinate is unchanged). Let $(M(x, y, z), N(x, y, z), (P(x, y, z)))$ be the components of a vector field \mathbf{F} in the original (x, y, z) coordinate. Express the first two components of \mathbf{F} , (M', N') in the new (x', y', z) coordinate in terms of M and N (P remains unchanged). The same formula also works for the normal vector $\mathbf{n} = (n_1, n_2, n_3)$ and the tangent vector $\mathbf{T} = (T_1, T_2, T_3)$
- (g) Show by direct calculation that

$$\begin{vmatrix} n'_1 & n'_2 & n_3 \\ \frac{\partial}{\partial x'} & \frac{\partial}{\partial y'} & \frac{\partial}{\partial z} \\ M' & N' & P \end{vmatrix} = \begin{vmatrix} n_1 & n_2 & n_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

and

$$T_1 F_1 + T_2 F_2 + T_3 F_3 = T'_1 F'_1 + T'_2 F'_2 + T_3 F_3$$

With the identities above, one can then perform a few successive rotations to transform a triangle in \mathbb{R}^3 into a triangle in $x - y$ plan. Therefore reduces Stoke's Theorem on a triangle to Green's Theorem in \mathbb{R}^2 . The latter can be easily verified via Fundamental Theorem of Calculus.