Homework Assignment for Week 04

Assigned Mar 19, 2009.

1. Section 9.8: Problems 9, 13, 28, 29, 31, 32, 33, 34(a), 35, 43.
2. Chap 9: Problems 27, 28, 29, 30, 32, 37, 47(a), 49-54(important), 71, 72.
3. Section 10.4: Problems 9, 13, 17, 23, 28, 29.
4. Solve the differential equation

$$dy/dx = 1 + y^2, \quad y(0) = 0,$$

by power series expansion. The differential equation can also be integrated directly. Verify your answer by comparing the first 2 nonzero coefficients of the Taylor series expansion of the solution, which can be found on p624.

5. Read p 619, which says the Taylor series generated by $(1+x)^m$, $m \in R$, converges on $|x| < 1$. It is not shown there that the series indeed converges to $(1+x)^m$. Analyzing $R_n(x)$ directly is not quite easy (try it and youll see why). An alternative approach is through the following steps:

   (a) Verify that

$$(k+1)\left(\begin{array}{c} m \\ k+1 \end{array}\right) + k\left(\begin{array}{c} m \\ k \end{array}\right) = m\left(\begin{array}{c} m \\ k \end{array}\right)$$

   (b) Define, for $|x| < 1$,

$$f(x) = \sum_{k=0}^{\infty} \left(\begin{array}{c} m \\ k \end{array}\right) x^k = 1 + mx + \frac{m(m-1)}{2!}x^2 \frac{m(m-1)(m-2)}{3!}x^3 + \cdots$$

   Show that $f(0) = 1$ and

$$(1+x)f'(x) = mf(x), \quad |x| < 1.$$

   (c) Show that $f(x) = (1+x)^m$ on $|x| < 1$.