# Homework Assignment for Chap 15

1. Section 15.1: Problems 9, 15, 17, 20, 25, 27, 28.

2. Section 15.2: Problems 5, 11, 17, 19, 21, 33 ('Work', 'Flow', 'Circulation' are alternative names for the integral $\int \boldsymbol{F} \cdot \boldsymbol{T} \, ds$, see Definition at p939 and p941. Also read Definition at p942 and then p943-944 for the definition and formula for 'flux' ).

3. Section 15.3: Problems 1, 3, 13, 19, 20, 23, 24, 30.

4. Section 15.4: Problems 3, 6, 15, 17, 21, 22, 26, 27, 29.

5. Section 15.5: Problems 3, 15, 19, 21, 33, 35.

6. Section 15.6: Problems 1, 15, 17, 19, 25, 27, 29, 37.

7. Section 15.7: Problems 1, 3, 7, 10, 13(a,c), 15, 16, 19.

8. This exercise is to show that Flux, Circulation and the Curl of a vector field does not depend on the coordinate you choose.

   Let $x'$, $y'$ be the coordinate axis obtained by rotating the $x$, $y$ axis by a fixed angle $\theta$.

   (a) Express $x'$, $y'$ in terms of $x$, $y$ and vice versa.

   (b) Express $\frac{\partial}{\partial x'}$, $\frac{\partial}{\partial y'}$ in terms of $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$ and vice versa.

   (c) Let $(M, N)$ be the components of a vector filed $\boldsymbol{F}$ in the original $(x, y)$ coordinate. Express the components of $\boldsymbol{F}$, $(M', N')$ in the new $(x', y')$ coordinates in terms of $M$ and $N$.

   (d) Use chain rule to verify that

   $$\frac{\partial N'}{\partial x'} - \frac{\partial M'}{\partial y'} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}$$

   and

   $$\frac{\partial M'}{\partial x'} + \frac{\partial N'}{\partial y'} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}$$

   (e) Express the unit vectors $\hat{x}'$, $\hat{y}'$ in terms of $\hat{x}$, $\hat{y}$ and vice versa.

   (f) Let $x'$, $y'$ be defined as above. In 3D, we perform the change of variable from $(x, y, z)$ to $(x', y', z)$ ($z$ coordinate is unchanged). Let $(M(x, y, z), N(x, y, z), (P(x, y, z))$ be the components of a vector field $\boldsymbol{F}$ in the original $(x, y, z)$ coordinate. Express the first two components of $\boldsymbol{F}$, $(M', N')$ in the new $(x', y', z)$ coordinate in terms of $M$ and $M$ ($P$ remains unchanged). The same formula also works for the normal vector $\boldsymbol{n} = (n_1, n_2, n_3)$ and the tangent vector $\boldsymbol{T} = (T_1, T_2, T_3)$

(g) Show by direct calculation that

$$
\begin{vmatrix} n'_1 & n'_2 & n_3 \\ \partial_{x'} & \partial_{y'} & \partial_z \\ M' & N' & P \end{vmatrix} = \begin{vmatrix} n_1 & n_2 & n_3 \\ \partial_x & \partial_y & \partial_z \\ M & N & P \end{vmatrix}
$$

and

$$ T_1 F_1 + T_2 F_2 + T_3 F_3 = T'_1 F'_1 + T'_2 F'_2 + T_3 F_3 $$

With the identities above, one can then perform a few successive rotations to transform a triangle lying in $\mathbb{R}^3$ into a triangle in $x - y$ plan, therefore reducing Stoke's Theorem on a triangle to Green's Theorem in $\mathbb{R}^2$. The latter can be easily verified via Fundamental Theorem of Calculus.

9. Section 15.8: Problems 1, 3, 13, 15, 17, 18, 19, 21, 23, 24.