Homework Assignment for Chap 06

1. Section 6.1: Problems 45, 47, 59, 65.
2. Section 6.2: Problems 28(c), 32, 45, 46.
3. Section 6.3: Problems 41, 50, 51, 57, 65, 77, 87, 89, 91, and as many as time permits in problems 31-40, 85-90.
4. Section 6.4: Problems: 9, 19, 25, 41, 47, 57, 71.
5. Evaluate $\lim_{n\to\infty}\sum_{k=\frac{n}{2}}^{n}\frac{1}{k}$. Hint: try to express it in terms of $\frac{k}{n}$.
6. Section 6.6: Problems: 17, 21, 27, 33, 37-46, 51, 54, 55.
7. (s6.6-extra1) Cauchy's Mean Value Theorem
   Prove the following variant of the Mean Value Theorem:
   Suppose f and g are continuous on $[a,b]$ and differentiable on (a,b), then there exists $c\in(a,b)$ such that
   $$\begin{vmatrix} f(b)-f(a) & f'(c) \\ g(b)-g(a) & g'(c) \end{vmatrix} = 0. \tag{1}$$
   Note that, this c need not satisfy $\dfrac{f(b)-f(a)}{b-a} = f'(c)$, nor $\dfrac{g(b)-g(a)}{b-a} = g'(c)$, but only $\dfrac{f(b)-f(a)}{g(b)-g(a)} = \dfrac{f'(c)}{g'(c)}$, which is the same as (1) provided $g(b)-g(a)\neq 0$.
   Hint: Apply standard Mean Value Theorem to
   $$F(x) = \begin{vmatrix} f(b)-f(a) & f(x)-f(a) \\ g(b)-g(a) & g(x)-g(a) \end{vmatrix} \qquad \text{on } [a,b].$$
8. (s6.6-extra2) Use Cauchy's Mean Value Theorem to prove the strong form of l'Hôpital's rule.
9. Section 6.7: Problems: 5, 7, 8, 11, 12, 17(a), 18.
10. Section 6.9: Problems: 9, 13, 21, 23, 31, 33, 39, 43, 57, 61, 67, 71, 79.
11. (s6.9-extra1) Show that
    $$\frac{d\csc^{-1}y}{dy} = \frac{-1}{|y|\sqrt{y^2-1}}, \qquad |y|>1$$
    Explain why the negative sign is chosen. You will need to start from the 'restricted' domain of csc to see this.

12. Section 6.10: Problems: 15, 31, 43, 67.

    For Section 6.10, skip the inverse hyperbolic functions part and concentrate on hyperbolic functions and calculus related to them.

13. Section 6.11: Problems: 17, 21, 23, 27.

14. (s6.11-extra1) Verify that both $\sinh x$ are $\cosh x$ are solutions of $y'' = y$. Then solve for

$$y'' = y, \qquad y(0) = 1, \quad y'(0) = 2.$$

15. (s6.11-extra2) Verify that both e^{2x} and xe^{2x} are solutions of $y'' - 4y' + 4y = 0$, therefore so is the combination $a_1 e^{2x} + a_2 x e^{2x}$. This is an example of the multiple root case: $(\lambda - 2)^2 = 0$. You can either verify by direct differentiation, or try to look for solutions of the form $z(x)e^{2x}$ and find that this leads to $z'' = (y(x)e^{-2x})'' = 0$.

16. (s6.11-extra3) Verify by direct differentiation that

$$\frac{d}{dx}(\cos(kx) + i\sin(kx)) = ik(\cos(kx) + i\sin(kx))$$

    This is a good explanation why one defines $\exp(ikx)$ to be $\cos(kx) + i\sin(kx)$.