Midterm Exam 2

May 09, 2013, 10:10 AM. Show all details.

1. (20 pts) Let $f(x,y) = \dfrac{x^2 y}{x^2+y^2}$, $P = (0,0)$ and $\boldsymbol{u}^\theta = (\cos\theta, \sin\theta)$, $\theta \in [0, 2\pi]$.

   (a) For fixed θ, write down the definition of the directional derivative $\left(\dfrac{df}{ds}\right)_{\boldsymbol{u}^\theta, P}$ and evaluate it.

   (b) Does f have a linear approximation near $(0,0)$? Explain.

2. (12 pts) Use Lagrangian multipliers (and only Lagrangian multipliers) to find extreme values of $f(x,y,z) = xy + 2z^2$ on

$$\begin{cases} x^2+y^2+z^2 = 9 \\ x - y = 0 \end{cases}$$

3. (16 pts)

   (a) State (need not prove) the second derivative test (discriminant test) for $f(x,y)$. You may assume that f and all its first and second derivatives are continuous in R^2.

   (b) Give an example of a function $g(x,y,z)$ which has a saddle point at $(0,0,0)$.

4. (12 pts) Evaluate

$$\int_0^2 \int_y^2 x^2 \cos(xy)\, dxdy$$

5. (12 pts) Switch the order of integration of

$$\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dzdydx$$

   to $dydxdz$ and $dxdydz$ respectively. You don't need to find the numerical value of the integral.

6. (16 pts) Express the volume of $\{\rho \le 1\} \cap \{\phi \le \pi/4\}$ as triple integrals in cylindrical and spherical coordinates, respectively. Then find the volume.

7. (12 pts) Find the Taylor expansion of $f(x,y,z)$ around (x_0, y_0, z_0) up to quadratic terms of x, y and z. Give an expression of the remainder term, $R_2 = \frac{1}{3!}(\cdots)$. You may assume that f and all its first and second derivatives are continuous in R^3.