§ 7.4

19. $y = x^{\frac{3}{2}}$ from $x = 0$ to $x = 4$

$\frac{dy}{dx} = \frac{3}{2} \cdot x^{\frac{1}{2}}$

$\therefore L = \int_0^4 \sqrt{1 + (\frac{3}{2}x^{\frac{1}{2}})^2}\, dx = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$

Let $u = 1 + \frac{9}{4}x \Rightarrow du = \frac{9}{4}dx \Rightarrow \frac{4}{9}du = dx$

$u(0) = 1 + \frac{9}{4} \cdot 0 = 1$

$u(4) = 1 + \frac{9}{4} \cdot 4 = 10$

$\therefore L = \int_1^{10} \sqrt{u} \cdot \frac{4}{9}\, du = \frac{4}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{10} = \frac{8}{27}(10\sqrt{10} - 1)$

28. $L = \int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

$\therefore (\frac{dy}{dx})^2 = \frac{1}{x^2} \Rightarrow \frac{dy}{dx} = \pm\frac{1}{x}$

So, $y = \pm \ln|x| + C$

If λ $(1, 0)$, $0 = \pm \ln 1 + C \Rightarrow C = 0$

Hence, $y = \pm \ln|x|$ *

46. $y = \sqrt{r^2 - x^2}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \frac{-2x}{\sqrt{r^2 - x^2}} = \frac{-x}{\sqrt{r^2 - x^2}}$

$\therefore (\frac{dy}{dx})^2 = \frac{x^2}{r^2 - x^2}$

$S = \int_a^{a+h} 2\pi \cdot y \cdot \sqrt{1 + (\frac{dy}{dx})^2}\, dx$

$= \int_a^{a+h} 2\pi \cdot \sqrt{r^2 - x^2} \cdot \sqrt{1 + \frac{x^2}{r^2 - x^2}}\, dx$

$= 2\pi \int_a^{a+h} \sqrt{r^2 - x^2} \cdot \sqrt{\frac{r^2 - x^2 + x^2}{r^2 - x^2}}\, dx$

$= 2\pi r \int_a^{a+h} 1\, dx = 2\pi r \cdot h$ which is independent of a.

56. $y = \frac{1}{2}\cosh 2x$ from $x = 0$ to $x = \ln\sqrt{5}$

$\frac{dy}{dx} = \sinh 2x \Rightarrow \left(\frac{dy}{dx}\right) = (\sinh 2x)^2$

$$L = \int_0^{\ln\sqrt{5}} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{\ln\sqrt{5}} \sqrt{1+(\sinh 2x)^2}\, dx = \int_0^{\ln\sqrt{5}} \sqrt{(\cosh 2x)^2}\, dx = \int_0^{\ln\sqrt{5}} |\cosh 2x|\, dx$$

$$= \int_0^{\ln\sqrt{5}} \cosh 2x \cdot dx = \frac{1}{2}\int_0^{\ln\sqrt{5}} \cosh 2x \cdot d2x = \frac{1}{2}\sinh 2x \Big|_0^{\ln\sqrt{5}}$$

$$= \frac{1}{2}\left[\sinh(2\ln\sqrt{5}) - \sinh 0\right] = \frac{1}{2}\left[\frac{5-\frac{1}{5}}{2} - 0\right] = \frac{6}{5}$$

58. $y = 2\cosh\left(\frac{x}{2}\right)$, $0 \le x \le \ln 8$, about the x-axis

$\frac{dy}{dx} = \sinh\left(\frac{x}{2}\right) \Rightarrow \left(\frac{dy}{dx}\right)^2 = \left(\sinh\left(\frac{x}{2}\right)\right)^2$

$$S = \int_0^{\ln 8} 2\pi \cdot y \cdot \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^{\ln 8} 2\pi \cdot 2\cosh\left(\frac{x}{2}\right) \cdot \sqrt{1+\left(\sinh\left(\frac{x}{2}\right)\right)^2}\, dx$$

$$= \int_0^{\ln 8} 4\pi \cosh\left(\frac{x}{2}\right) \cdot \cosh\left(\frac{x}{2}\right) dx$$

$$= 4\pi \int_0^{\ln 8} \left(\cosh\frac{x}{2}\right)^2 dx$$

$$= 4\pi \int_0^{\ln 8} \left(\frac{e^{\frac{x}{2}} + e^{-\frac{x}{2}}}{2}\right)^2 dx = \pi \int_0^{\ln 8} \left(e^x + 2 + e^{-x}\right) dx$$

$$= \pi\left(e^x + 2x - e^{-x}\right)\Big|_0^{\ln 8} = \pi\left(8 + 2\ln 8 - \frac{1}{8}\right)$$

$$= \pi\left(\frac{63}{8} + 2\ln 8\right)$$