§ 6.9

52. $\int \frac{dx}{\sqrt{2x-x^2}}$

$= \int \frac{dx}{\sqrt{1-(x^2-2x+1)}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1)+C$

54. $\int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{3+4t-4t^2}}$

$= \int_{\frac{1}{2}}^{1} \frac{6\,dt}{\sqrt{4-(4t^2-4t+1)}} = \int_{\frac{1}{2}}^{1} \frac{3\,dt}{\sqrt{1-(\frac{2t-1}{2})^2}} = 3\sin^{-1}(t-\frac{1}{2})\Big|_{\frac{1}{2}}^{1}$

$= 3\sin^{-1}(\frac{1}{2}) - 3\sin^{-1}(0) = 3\cdot\frac{\pi}{6} - 3\cdot 0 = \frac{\pi}{2}$

56. $\int \frac{dy}{y^2+6y+10}$

$= \int \frac{dy}{1+(y+3)^2} = \tan^{-1}(y+3)+C$

58. $\int_{2}^{4} \frac{2\,dx}{x^2-6x+10}$

$= \int_{2}^{4} \frac{2\,dx}{1+(x-3)^2} = 2\tan^{-1}(x-3)\Big|_{2}^{4} = 2\tan^{-1}(1) - 2\tan^{-1}(-1)$

$= 2\cdot\frac{\pi}{4} - 2\cdot(-\frac{\pi}{4}) = \pi$

64. $\int \frac{\sqrt{\tan^{-1}x}\,dx}{1+x^2}$

令 $u = \tan^{-1}x \Rightarrow du = \frac{1}{1+x^2}dx$

原式 $= \int \sqrt{u}\,du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(\tan^{-1}x)^{\frac{3}{2}} + C$

68. $\int_{1}^{2} \frac{\cos(\sec^{-1}x)\,dx}{x\sqrt{x^2-1}}$

令 $u = \sec^{-1}x \Rightarrow du = \frac{1}{|x|\sqrt{x^2-1}}dx$

$\because x \in [1,2] \Rightarrow du = \frac{1}{x\sqrt{x^2-1}}dx$

$u(1) = \sec^{-1}(1) = 0$

$u(2) = \sec^{-1}(2) = \frac{\pi}{3}$

原式 $= \int_{0}^{\frac{\pi}{3}} \cos u\,du = \sin u\Big|_{0}^{\frac{\pi}{3}} = \frac{\sqrt{3}}{2}$

8b.

(a) True!

$$\left(-\cos^{-1}x + C\right)' = -\frac{-1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

(b) True!

$$\left(\cos^{-1}(-x) + C\right)' = \frac{-1}{\sqrt{1-(-x)^2}} \cdot (-1) = \frac{1}{\sqrt{1-x^2}}$$