§6.4

10. $\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\,dr$

令 $u=-\sqrt{r} \Rightarrow du=-\frac{1}{2}\frac{1}{\sqrt{r}}dr \Rightarrow -2\,du=\frac{1}{\sqrt{r}}dr$

原式 $=\int e^{u}\cdot(-2\,du)=-2e^{u}+C=-2e^{-\sqrt{r}}+C$

19. $\int_{\ln(\frac{\pi}{6})}^{\ln(\frac{\pi}{2})} 2e^{v}\cos e^{v}\,dv$

令 $u=e^{v} \Rightarrow du=e^{v}\cdot dv$

$u(\ln(\frac{\pi}{6}))=e^{\ln\frac{\pi}{6}}=\frac{\pi}{6}$

$u(\ln(\frac{\pi}{2}))=e^{\ln\frac{\pi}{2}}=\frac{\pi}{2}$

原式 $=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 2\cos u\,du=2\sin u\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=2\sin\frac{\pi}{2}-2\sin\frac{\pi}{6}=2-1=1$

26. $\int_{1}^{4}\frac{2^{\sqrt{x}}}{\sqrt{x}}\,dx$

令 $u=\sqrt{x} \Rightarrow du=\frac{1}{2}\cdot\frac{1}{\sqrt{x}}dx \Rightarrow 2\,du=\frac{1}{\sqrt{x}}dx$

$u(1)=\sqrt{1}=1$

$u(4)=\sqrt{4}=2$

原式 $=\int_{1}^{2}2^{u}\cdot 2\,du=2\cdot\frac{1}{\ln 2}\cdot 2^{u}\Big|_{1}^{2}=\frac{4}{\ln 2}$

48. $\int_{\frac{5}{6}}^{1}\pi\tan\pi x\,dx$

$=\int_{\frac{5}{6}}^{1}\pi\frac{\sin\pi x}{\cos\pi x}\,dx$

令 $u=\cos\pi x \Rightarrow du=-\pi\sin\pi x\,dx \Rightarrow -du=\pi\sin\pi x\,dx$

$u(\frac{5}{6})=\cos(\pi\frac{5}{6})=-\frac{\sqrt{3}}{2}$

$u(1)=\cos(\pi)=-1$

原式 $=\int_{-\frac{\sqrt{3}}{2}}^{-1}\frac{1}{u}\cdot -du=-\ln u\Big|_{-\frac{\sqrt{3}}{2}}^{-1}=-\ln\frac{-1}{-\frac{\sqrt{3}}{2}}=-\ln\frac{2}{\sqrt{3}}$

70. $\frac{dy}{dt} = e^{-t}\sec^2(\pi e^{-t})$

$y = \frac{2}{\pi}$ when $t = \ln 4$

$y = \int e^{-t}\sec^2(\pi e^{-t})\,dt$

令 $u = \pi e^{-t} \Rightarrow du = -\pi e^{-t}dt \Rightarrow -\frac{1}{\pi}du = e^{-t}dt$

$y = \int \sec^2 u \cdot (-\frac{1}{\pi}du) = -\frac{1}{\pi}\tan u + C = -\frac{1}{\pi}\tan(\pi e^{-t}) + C$

$\because y = \frac{2}{\pi}$ when $t = \ln 4$

$\therefore \frac{2}{\pi} = -\frac{1}{\pi}\tan(\pi e^{-\ln 4}) + C \Rightarrow \frac{2}{\pi} = -\frac{1}{\pi}\tan(\pi e^{\ln 4^{-1}}) + C$

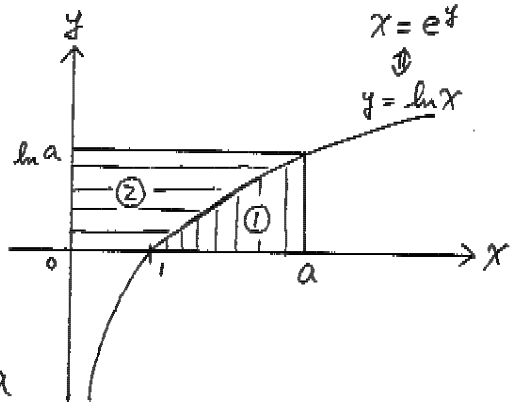
$\Rightarrow \frac{2}{\pi} = -\frac{1}{\pi}\tan(\frac{\pi}{4}) + C \Rightarrow C = \frac{3}{\pi}$

So, $y = -\frac{1}{\pi}\tan(\pi e^{-t}) + \frac{3}{\pi}$

78. $\int_1^a \ln x\,dx + \int_0^{\ln a} e^y dy = a\ln a$

$\int_1^a \ln x\,dx = ①$

$\int_0^{\ln a} e^y dy = ②$



$\therefore \int_1^a \ln x\,dx + \int_0^{\ln a} e^y dy = ① + ② = a\cdot\ln a$

79.

(a) $(x\ln x - x + c)' = \ln x + x\cdot\frac{1}{x} - 1 = \ln x$

So, $\int \ln x\,dx = x\ln x - x + C$

(b) $\frac{1}{e-1}\int_1^e \ln x\,dx = \frac{1}{e-1}\left[x\ln x - x\right]\Big|_1^e$

$= \frac{1}{e-1}\left[\underbrace{(e\cdot\ln e - e)}_{=0} - \underbrace{(1\cdot\ln 1 - 1)}_{=-1}\right]$

$= \frac{1}{e-1}$