§5.6

53. (a) $\int_0^{2\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\,dz$

令 $u = 4+3\sin z \Rightarrow du = 3\cos z\,dz$

$\therefore u(0) = 4+3\sin 0 = 4$

$u(2\pi) = 4+3\sin 2\pi = 4$

原式 $= \int_4^4 \frac{1}{\sqrt{u}} \cdot \frac{1}{3}\,du = 0$

(b) $\int_{-\pi}^{\pi} \frac{\cos z}{\sqrt{4+3\sin z}}\,dz$

令 $u = 4+3\sin z \Rightarrow du = 3\cos z\,dz$

$\therefore u(-\pi) = 4+3\sin -\pi = 4$

$u(\pi) = 4+3\sin \pi = 4$

原式 $= \int_4^4 \frac{1}{\sqrt{u}} \cdot \frac{1}{3}\,du = 0$

54. (a) $\int_{-\frac{\pi}{2}}^{0} \frac{\sin w}{(3+2\cos w)^2}\,dw$

令 $u = 3+2\cos w \Rightarrow du = -2\sin w\,dw$

$\Rightarrow -\frac{1}{2}du = \sin w\,dw$

$u(-\frac{\pi}{2}) = 3+2\cos(-\frac{\pi}{2}) = 3$

$u(0) = 3+2\cos(0) = 5$

原式 $= \int_3^5 \frac{1}{u^2} \cdot -\frac{1}{2}\,du = \frac{1}{2}\cdot\frac{1}{u}\Big|_3^5$

$= \frac{1}{2}\left(\frac{1}{5}-\frac{1}{3}\right) = -\frac{1}{15}$

(b) $\int_0^{\frac{\pi}{2}} \frac{\sin w}{(3+2\cos w)^2}\,dw$

令 $u = 3+2\cos w \Rightarrow du = -2\sin w\,dw$

$\Rightarrow -\frac{1}{2}du = \sin w\,dw$

$u(0) = 3+2\cos(0) = 5$

$u(\frac{\pi}{2}) = 3+2\cos(\frac{\pi}{2}) = 3$

原式 $= \int_5^3 \frac{1}{u^2} \cdot -\frac{1}{2}\,du = \frac{1}{2}\cdot\frac{1}{u}\Big|_5^3$

$= \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{1}{15}$

56. $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2}$

令 $u = 1+\sqrt{y} \Rightarrow du = \frac{1}{2}\cdot\frac{1}{\sqrt{y}}dy$

$u(1) = 1+\sqrt{1} = 2$

$u(4) = 1+\sqrt{4} = 3$

原式 $= \int_2^3 \frac{1}{u^2}du = -\frac{1}{u}\Big|_2^3 = -\left(\frac{1}{3}-\frac{1}{2}\right) = \frac{1}{6}$

58. $\int_{\pi}^{\frac{3\pi}{2}} \tan^{-5}\left(\frac{\theta}{6}\right)\sec^2\left(\frac{\theta}{6}\right)d\theta$

令 $u = \tan\left(\frac{\theta}{6}\right) \Rightarrow du = \frac{1}{6}\sec^2\left(\frac{\theta}{6}\right)d\theta$

$\Rightarrow 6\,du = \sec^2\left(\frac{\theta}{6}\right)d\theta$

$u(\pi) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

$u\left(\frac{3\pi}{2}\right) = \tan\left(\frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

原式 $= \int_{\frac{1}{\sqrt{3}}}^{1} u^{-5}\cdot 6\,du = 6\cdot\frac{u^{-4}}{-4}\Big|_{\frac{1}{\sqrt{3}}}^{1} = -\frac{3}{2}\left(1-9\right) = 12$

14. $\int \sqrt{1+\sin^2(x-1)}\,\sin(x-1)\cos(x-1)\,dx$

(a) 令 $u = x-1$, $du = dx$

原式 $= \int \sqrt{1+\sin^2 u}\,\sin u\cos u\,du$ ···· ①

令 $v = \sin u$, $dv = \cos u\,du$

① $= \int \sqrt{1+v^2}\,v\,dv$ ···· ②

令 $w = 1+v^2$, $dw = 2v\,dv$

② $= \int \sqrt{w}\cdot\frac{1}{2}dw = \frac{1}{2}\cdot\frac{w^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{3}w^{\frac{3}{2}} + C = \frac{1}{3}\left(1+\sin^2(x-1)\right)^{\frac{3}{2}} + C$

(b) 令 $u = \sin(x-1)$, $du = \cos(x-1)\,dx$

原式 $= \int \sqrt{1+u^2}\cdot u\cdot du$ ···· ③

令 $v = 1+u^2$, $dv = 2u\,du$

③ $= \int \sqrt{v}\cdot\frac{1}{2}dv = \frac{1}{3}v^{\frac{3}{2}} + C = \frac{1}{3}\left(1+\sin^2(x-1)\right)^{\frac{3}{2}} + C$

(c) 令 $u = 1+\sin^2(x-1)$, $du = 2\sin(x-1)\cos(x-1)\,dx$

原式 $= \int \sqrt{u}\cdot\frac{1}{2}du = \frac{1}{3}u^{\frac{3}{2}} + C = \frac{1}{3}\left(1+\sin^2(x-1)\right)^{\frac{3}{2}} + C$