§5.5

8. $\int \left(\frac{1}{5} - \frac{2}{x^3} + 2x\right) dx$

$= \int \left(\frac{1}{5} - 2x^{-3} + 2x\right) dx = \frac{1}{5}x - 2 \cdot \frac{x^{-2}}{-2} + x^2 + C = \frac{1}{5}x + \frac{1}{x^2} + x^2 + C$

10. $\int x^{-\frac{5}{4}} dx$

$= \frac{x^{-\frac{1}{4}}}{-\frac{1}{4}} + C = -4x^{-\frac{1}{4}} + C$

12. $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}}\right) dx$

$= \int \left(\frac{1}{2}x^{\frac{1}{2}} + 2 \cdot x^{-\frac{1}{2}}\right) dx = \frac{1}{2} \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2 \cdot \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{1}{3}x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + C$

14. $\int \left(\frac{1}{7} - \frac{1}{y^{\frac{5}{4}}}\right) dy$

$= \int \left(\frac{1}{7} - y^{-\frac{5}{4}}\right) dy = \frac{1}{7}y - \frac{y^{-\frac{1}{4}}}{-\frac{1}{4}} + C = \frac{1}{7}y + 4y^{-\frac{1}{4}} + C$

24. $\int \left(-\frac{\sec^2 x}{3}\right) dx$

$= -\frac{1}{3} \cdot \tan x + C$

45. (a) False

$\left(\frac{x^2}{2}\sin x + C\right)' = x\sin x + \frac{x^2}{2}\cos x$

(b) False

$(-x\cos x + C)' = -\cos x + x\sin x$

(c) True!

$(-x\cos x + \sin x + C)' = -\cos x + x\sin x + \cos x$

$= x\sin x$

50. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, $y = 0$ when $x = 4$

$y = \int \frac{1}{2\sqrt{x}}\, dx = \sqrt{x} + C$

$\because$ when $x = 4$, $y = 0$

$\therefore \quad 0 = \sqrt{4} + C \quad \Rightarrow C = -2$

So. $y = \sqrt{x} - 2$

54. $\frac{d^2y}{dx^2} = \frac{2}{x^3}$, $\frac{dy}{dx} = -1$ and $y = \frac{1}{2}$ when $x = \frac{1}{2}$

$\frac{dy}{dx} = \int \frac{2}{x^3}\, dx = 2 \cdot \frac{x^{-2}}{-2} + C_1 = -x^{-2} + C_1$

$\because \frac{dy}{dx} = -1$ when $x = \frac{1}{2}$

$\therefore \quad -1 = -(\frac{1}{2})^{-2} + C_1 \quad \Rightarrow C_1 = 3$

So. $\frac{dy}{dx} = -\frac{1}{x^2} + 3$

$y = \int (-\frac{1}{x^2} + 3)\, dx = x^{-1} + 3x + C_2$

$\because y = \frac{1}{2}$ when $x = \frac{1}{2}$

$\therefore \quad \frac{1}{2} = 2 + \frac{3}{2} + C_2 \quad \Rightarrow C_2 = 0$

Hence,

$y = \frac{1}{x} + 3x$