§5.4

6. $\int_0^5 x^{\frac{3}{2}}\,dx$

$$\int_0^5 x^{\frac{3}{2}}\,dx = \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\Big|_0^5 = \frac{2}{5}\,x^{\frac{5}{2}}\Big|_0^5 = \frac{2}{5}\cdot 5^{\frac{5}{2}} = 10\sqrt{5}$$

12. $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \csc^2 x\,dx = -\cot x\Big|_{\frac{\pi}{6}}^{\frac{5}{6}\pi} = -\cot\frac{5\pi}{6} + \cot\frac{\pi}{6} = -(-\sqrt{3}) - (-\sqrt{3}) = 2\sqrt{3}$

14. $\int_0^{\frac{\pi}{3}} 4\sec u\tan u\,du = 4\sec u\Big|_0^{\frac{\pi}{3}} = 4\sec\frac{\pi}{3} - 4\sec 0 = 4\cdot 2 - 4\cdot 1 = 4$

20. $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4)\,dt = \int_{-\sqrt{3}}^{\sqrt{3}} (t^3+t^2+4t+4)\,dt = \left(\frac{t^4}{4} + \frac{t^3}{3} + 2t^2 + 4t\right)\Big|_{-\sqrt{3}}^{\sqrt{3}}$

$$= 2\cdot\frac{(\sqrt{3})^3}{3} + 2\cdot 4\cdot\sqrt{3} = 10\sqrt{3}$$

56. (a) $V = \frac{ds}{dt} = \frac{d}{dt}\int_0^t g(x)\,dx = g(t)$

$\Rightarrow V(3) = g(3) = 0$

(b) $a = \frac{dV}{dt} = g'(t)$

$a(3) = g'(3) > 0$. Positive !

(c) At $t=3$, the particle position is $\int_0^3 g(x)\,dx = -\frac{3\times 6}{2} = -9$

(d) $\because s(6) = \int_0^6 g(x)\,dx = -9 + 9 = 0$

$\therefore$ The particle passes through the origin at $t=6$

(e) $a = \frac{dV}{dt} = g'(t)$

$a(1) = g'(1) = 0$, At $t=1$!

(f) The particle starts at the origin and moves away to the left for $0<t<3$.
It moves back toward the origin for $3<t<6$, passes through the origin at $t=6$, and moves away to the right for $t>6$.

(g) Right side , $\because \int_0^9 g(x)\,dx > 0$

66. $\int_0^h \pi\left(\frac{r}{h}x\right)^2 dx = \pi\cdot\frac{r^2}{h^2}\int_0^h x^2\,dx = \pi\cdot\frac{r^2}{h^2}\cdot\frac{x^3}{3}\Big|_0^h = \frac{1}{3}\pi r^2 h$

70. Find $f(4)$ if $\int_0^x f(t)\,dt = x\cos\pi x$

$f(x) = \frac{d}{dx}\int_0^x f(t)\,dt = \cos\pi x - \pi x\sin(\pi x)$

$\Rightarrow f(4) = \cos(\pi\cdot 4) - \pi\cdot 4\sin(\pi\cdot 4) = 1$

74. $h(x) = \int_0^x f(t)\,dt$

(a) True. $\because h'(x) = f(x) \Rightarrow h''(x) = f'(x)$

Since f is differentiable for all x,

h has a second derivative for all x.

(b) True. 可微 $\Rightarrow$ cont'i.

$\because h'(x) = f(x) \Longrightarrow h(x)$ 可微

$h''(x) = f'(x) \Longrightarrow h'(x)$ 可微

So, $h(x)$ & $\frac{dh}{dx}$ are both cont'i.

(c) True. $\because h'(1) = f(1) = 0$

(d) True. $\because h'(1) = f(1) = 0 \Rightarrow x=1$ is a critical point

& $h''(1) = f'(1) < 0$, So h has a local max at $x=1$

(e) False.

(f) False, $\because h''(1) = f'(1) < 0$ never changes sign.

(g) True, $\because h'(1) = f(1) = 0$

& $h''(x) = f'(x) < 0 \Rightarrow$ 表 $h'(x)$ is decreasing

$\therefore$ The graph of $\frac{dh}{dx}$ crosses the x-axis at $x=1$