10/29 材料華 習題詳解

微積分

§ 3.6

22. $(3xy+1)^2 = 6y$, Find $\frac{dy}{dx}$

$2(3xy+1)\cdot\left(3y + 3x\frac{dy}{dx}\right) = 6\frac{dy}{dx}$

$6y(3xy+1) + 6x(3xy+1)\frac{dy}{dx} = 6\frac{dy}{dx}$

So, $\left[6x(3xy+1)-6\right]\frac{dy}{dx} = -6y(3xy+1)$

$\therefore \frac{dy}{dx} = \frac{y(3xy+1)}{1-x(3xy+1)}$ #

26. $x = \sin y$, Find $\frac{dy}{dx}$

$1 = \cos y \cdot \frac{dy}{dx}$

So, $\frac{dy}{dx} = \frac{1}{\cos y} = \sec y$ #

34. $\cos r + \cos\theta = r\theta$, Find $\frac{dr}{d\theta}$

$-\sin r\cdot\frac{dr}{d\theta} - \sin\theta = r + \theta\frac{dr}{d\theta}$

So, $\frac{dr}{d\theta} = \frac{r+\sin\theta}{-\theta-\sin r}$ #

38. $y^2 + 2y = 2x + 1$, Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$

(i) $2y\cdot\frac{dy}{dx} + 2\cdot\frac{dy}{dx} = 2 \Rightarrow (y+1)\frac{dy}{dx} = 1$

$\therefore \frac{dy}{dx} = \frac{1}{y+1}$ #

(ii) $\because (y+1)\frac{dy}{dx} = 1$

$\therefore \frac{dy}{dx}\cdot\frac{dy}{dx} + (y+1)\cdot\frac{d^2y}{dx^2} = 0$

So, $\frac{d^2y}{dx^2} = \frac{-\left(\frac{dy}{dx}\right)^2}{y+1}$

$= \frac{-\left(\frac{1}{y+1}\right)^2}{y+1} = -\frac{1}{(y+1)^3}$ #

44. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$

$2y\cdot\frac{dy}{dx} - 2 - 4\cdot\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{1}{y-2}$

(a) the slope of the tangent line $m = \left.\frac{dy}{dx}\right|_{(-2,1)} = \left.\left(\frac{1}{y-2}\right)\right|_{(-2,1)} = -1$

the tangent line is $y - 1 = -1(x+2)$ #

(b) the normal line is $y - 1 = 1\cdot(x+2)$ #

§ 3.6

48. $a = \frac{dv}{dt} = \frac{d}{dt}\left(k\sqrt{s}\right) = \frac{k}{2\sqrt{s}} \cdot \underbrace{\frac{ds}{dt}}_{\| \; v = k\sqrt{s}} = \frac{k}{2\sqrt{s}} \cdot k\sqrt{s} = \frac{k^2}{2}$

So, the acceleration is constant. *

§ 3.7

20. $(1+x)^k \approx 1 + kx$

(a) $(1.0002)^{50} \approx 1 + 50 \times 0.0002 = 1.01$

(b) $\sqrt[3]{1.009} = (1.009)^{\frac{1}{3}} \approx 1 + \frac{1}{3} \times 0.009 = 1.003$ #

34. $S = 6x^2$

$S_{x_0+dx} - S_{x_0} = 6(x_0+dx)^2 - 6x_0^2 = 12x_0\,dx + 6(dx)^2$

51. If f has horizontal tangent at $x=a$, then $f'(a) = 0$

So, the linearization of f at $x=a$ is $y - f(a) = f'(a)\cdot(x-a)$

Hence, $y = f(a)$. The linearization is a constant.

§ 3.8

11. $f(x) = \begin{cases} \sqrt{x}, & x \geq 0 \\ \sqrt{-x}, & x < 0 \end{cases}$

(i) If $x_0 = h > 0$

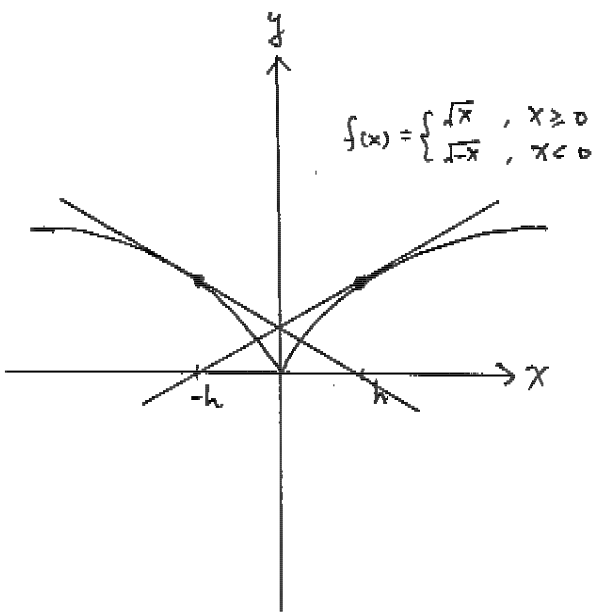
$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = h - \frac{f(h)}{f'(h)}$$

$$= h - \frac{\sqrt{h}}{\frac{1}{2\sqrt{h}}} = h - 2h = -h$$

(ii) If $x_0 = -h$, $h > 0$

$$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -h - \frac{f(-h)}{f'(-h)}$$

$$= -h - \frac{\sqrt{h}}{\frac{-1}{2\sqrt{h}}} = -h + 2h = h$$



§3.8

12. $f(x) = x^{\frac{1}{3}}$ with $x_0 = 1$

$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n^{\frac{1}{3}}}{\frac{1}{3}x_n^{-\frac{2}{3}}}$$

$$= x_n - 3x_n = -2x_n$$

So, $x_1 = -2x_0 = -2$

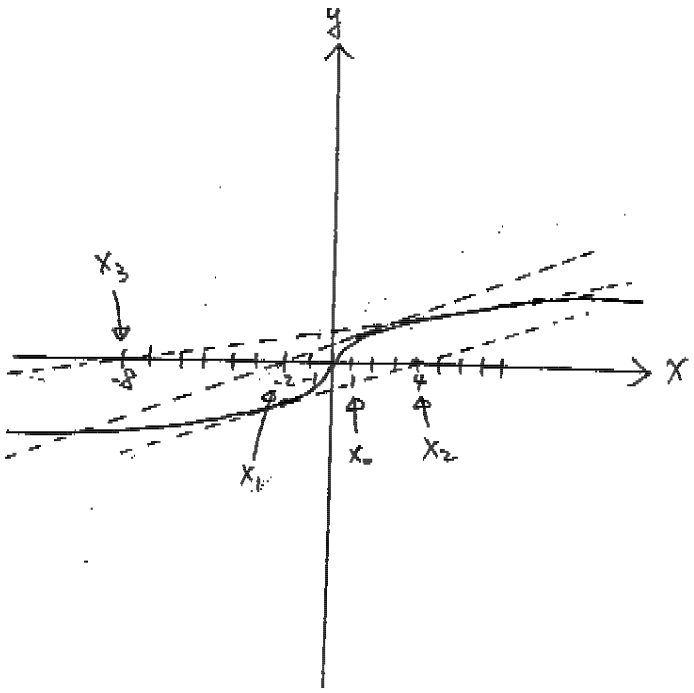
$x_2 = -2x_1 = 4$

$x_3 = -2x_2 = -8$

$x_4 = -2x_3 = 16$

Since $|x_n| = 2x_{n-1} = 2^n$.

$\therefore$ When $n \to \infty \Rightarrow |x_n| \to \infty$



22. $\because \frac{1}{x^2+1} = x$ ←

$\therefore x^3 + x - 1 = 0$

let $f(x) = x^3 + x - 1$

$\therefore f'(x) = 3x^2 + 1$

So, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{x_n^3 + x_n - 1}{3x_n^2 + 1}$$

Hence,

$x_0 = 1$

$x_1 = 0.75$

$x_2 = 0.686046511\,6$

$x_3 = 0.682339582\,6$

$x_4 = 0.682327803\,9$

$x_5 = 0.682327803\,8$

Therefore, 0.68233 is five decimal places.

$d^2 = (x-2)^2 + (x^2 + \frac{1}{2})^2$

$= x^2 - 4x + 4 + x^4 + x^2 + \frac{1}{4}$

$= x^4 + 2x^2 - 4x + \frac{17}{4}$

令 $(d^2)' = 0$, find x ⇐ 極值所在

$4x^3 + 4x - 4 = 0$

$x^3 + x - 1 = 0$

Find x, 即在 $y = x^2$ 上距離值 $(2, -\frac{1}{2})$ 最近的點!

chap 3

90. let $f(x) = \frac{1}{1+\tan x}$

$$f'(x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

$\therefore f(0) = 1$, $f'(0) = -1$

So, the linearization at $x = 0$ is

$$y = f(0) + f'(0)\cdot(x-0)$$
$$= 1 - x \quad \#$$