Midterm

Tue April 22, 2003, 10:10 AM. SHOW ALL YOUR WORK.

1. (10 pts) For what value of $p \in R$ is the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ convergent? Explain.

2. (10 pts) Give an example of a power series that converges on $[3, 5)$ and diverges elsewhere. Explain.

3. (a) (5 pts) State (need not prove) Taylor's Theorem (You can ignore the assumptions on the function and simply write down n terms approximation and the formula of the remainder term).

   (b) (10 pts) Give an approximate value of $\sqrt{9.01}$ to 2 decimals. Explain why the error of your answer is $< 5 \cdot 10^{-3}$.

4. (a) (10 pts) Write down the Taylor series of $\exp(x)$ around $x = 1$ (you don't need to show that $\exp(x) =$ the Taylor series, just write down the series) and give the radius of convergence. Explain.

   (b) (5 pts) Find the sum of the series $1 - \frac{\pi^2}{4 \cdot 2!} + \frac{\pi^4}{16 \cdot 4!} - \cdots + (-1)^n \frac{\pi^{2n}}{2^{2n} \cdot (2n)!} + \cdots$.

   (c) (5 pts) Find the sum of the series $0.1 - \frac{0.01}{2} + \frac{0.001}{3} - \cdots + (-1)^{n-1} \frac{0.1^n}{n} + \cdots$.

5. Let a parametrized curve be given by $x = \cos t$, $y = t + \sin t$.

   (a) (5 pts) Find the slope of the curve at $t = \pi/4$.

   (b) (5 pts) Find the length of the curve on $0 \le t \le \pi/2$. Give your answer as a definite integral, but need not find the numerical value of the integral.

6. (10 pts) Write down the formula (need not prove) for the slope and length for the polar graph $r = f(\theta)$.

7. (10 pts) In each of the following, the coordinate in one of the 3 coordinate systems (rectangular (x, y, z), cylindrical (r, θ, z) and spherical (ρ, φ, θ)) is give. Write the coordinate in the other two coordinate systems. Restrict the answer to $r > 0, 0 \le \theta < 2\pi$ in cylindrical system and $\rho > 0, 0 \le \varphi \le \pi, 0 \le \theta < 2\pi$ in spherical system.

   (a) $(2, 0, 1)$ in cylindrical coordinate system.

   (b) $(1, \pi/2, \pi/2)$ in spherical coordinate system.

8. (25 pts) Find $\boldsymbol{T}$, $\boldsymbol{N}$, $\boldsymbol{B}$, κ and τ for the curve $x = \cos t$, $y = \sin t$, $z = t$.
   Show details of your work.